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A new fuzzy programming method to derive the priority vector from an interval reciprocal comparison matrix

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ABSTRACT

In this paper, we discuss the interval reciprocal comparison matrices and suggest a new fuzzy programming method (NFPM) to derive the priority vector from an interval reciprocal comparison matrix. Based on Mikhailov's membership function, we first give a membership function, which is used to measure the decision maker's satisfaction degree for the priority vector derived from each interval constraint. Then by using the membership function, we propose a new fuzzy programming method to get the most satisfaction degree of each priority vector. The NFPM has many prominent characteristics, especially, it is capable of deriving the priority vector from a consistent or inconsistent interval reciprocal comparison matrix. Moreover, the NFPM can overcome some drawbacks of Mikhailov's fuzzy programming method (FPM) for inconsistent interval reciprocal comparison matrices. Finally, several examples are given to illustrate the applicability and advantage of the proposed method.

1. Introduction

The Analytic Hierarchy Process (AHP) [25] is a powerful tool for dealing with decision making problems, such as planning, selecting a best alternative, resource allocations, and forecasting [32,33,45]. It mainly includes three parts: decomposition, measurement of preferences and synthesis. The key part of the AHP is measurement of preferences, in which the decision makers (DMs) construct pairwise comparison matrices to capture the relative importance degrees of objects (attributes or alternatives).

Since the DMs provide the pairwise comparison matrix based on historical data and their experiences, the pairwise comparison matrix is often inconsistent. To solve this problem, many scholars have done a lot of work for several decades [5,9,11,12,14,15,17,18,21,24,26,27,34,38,40,44]. Generally speaking, the approaches to this type of problems can be classified into three categories. The first category of approaches is to find a single inconsistent element and then adjust it. For example, Harker and Vargas [11] identified an element whose adjustment would result in the largest rate of change in the matrix's consistency level. Saaty [28] also discussed two similar approaches for revising a single element, both of which are based on the inconsistent pairwise comparison matrix and its principal eigenvector. The second category of approaches tends to modify several elements to improve the consistency level. For example, Xu and Wei [44] developed a consistent matrix

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by an auto-adaptive process based on the inconsistent matrix. The third category of approaches is to derive a priority vector by solving an optimization problem, such as the direct least square method and the weighted least square method [7].

The pairwise comparison matrices are a basic tool to express the DMs' preference information in the process of decision making. As we know that there are three kinds of different preference formats (i.e., multiplicative preference relations (MPRs, also known as reciprocal comparison matrices) [6], fuzzy preference relations (FPRs, or called complementary comparison matrices) [43] and linguistic preference relations (LPRs, or called linguistic comparison matrices) [42]). With the difficulty of giving a precise pairwise comparison matrix, it is reasonable to provide an interval comparison matrix for expressing the DM's uncertain preference information. Up to now, lots of research has focused on the problems of obtaining the priority weights from the interval complementary comparison matrices [1,8,19,37,39,46]. Due to the increasing complexity and uncertainty of the socio-economic environment, the problems with unbalanced distributions appear everywhere. Here we take the law of diminishing marginal utility in economics for example [41], if we invest the same resources to a company with bad performance and to a company with good performance respectively, the former yields more utility than the latter [20]. To deal with such a situation, interval reciprocal comparison matrices are a useful tool, especially in expressing uncertain multiplicative preference information.

Some approaches have been proposed to derive the priority vectors from the interval reciprocal comparison matrices. For example, Arbel [2] first used a linear programming model to get the priority weights of criteria from an interval reciprocal matrix. Kress [16] pointed out that Arbel's linear programming model cannot be used to deal with the inconsistent interval reciprocal comparison matrices. Arbel and Vargas [3] used the Monte Carlo simulation method and Arbel's linear programming model to deal with an interval reciprocal comparison matrix. They tried to establish a connection between the two approaches for the development of a more general approach. Islam et al. [13] introduced a lexicographic goal programming (LGP) model to derive the priority vector from an inconsistent interval reciprocal comparison matrix. However, Wang [35] proved that the LGP is defective in theory. In addition, Wang et al. [36] developed an eigenvector method-based nonlinear programming (NLP) approach to generate the interval weights and used a preference ranking method to compare the interval weights of criteria. Mikhailov [22,23] proposed a fuzzy programming method (FPM) to obtain the priority vector from an interval reciprocal comparison matrix.

From the above analysis, we can know that lots of work has been done on the interval reciprocal comparison matrices. However, due to the lack of information, it is difficult to get the priority vector from an interval reciprocal comparison matrix, especially, when the interval reciprocal comparison matrix given by the DM is not consistent. Up to now, some programming methods (such as the goal programming method and the fuzzy programming method) have been used to deal with this issue [13,22], but it is still an intractable problem. In this paper, we find that Mikhailov's FPM is defective. As we know, the upper and lower triangular judgments of an inconsistent interval reciprocal comparison matrix provide the same information on the preferences about weights. However, the priority vector derived from the upper triangular judgments is different from that derived from the lower triangular judgments by using the FPM. In order to overcome this drawback, in this paper, we propose a new fuzzy programming method (NFPM) to obtain the priority vector from an interval reciprocal comparison matrix. The NFPM is based on Mikhailov's FPM and determines the priority vector corresponding to the point with the most satisfaction degree. Moreover, the NFPM can also be reduced to a linear programming method.

The rest of the paper is organized as follows: Section 2 reviews some basic knowledge of reciprocal comparison matrix and consistent reciprocal comparison matrix. In Section 3, we recall Mikhailov's FPM and develop a new model to deal with an interval reciprocal comparison matrix. In Section 4, we give several examples to illustrate the effectiveness of the NFPM and compare it with some existing methods. Section 5 ends the paper with some conclusions.

2. Preliminaries

In what follows, we start by introducing some basic concepts and terminologies related to interval reciprocal comparison matrices. Suppose that a multi-criteria decision making problem consists of a set of alternatives $X = \{x_1, x_2, \ldots, x_m\}$ and a set of criteria $C = \{C_1, C_2, \ldots, C_n\}$. The DM evaluates each alternative with respect to the predefined criteria and then constructs a decision matrix using the given evaluation values, based on which the best alternative can be selected. In practice, it is difficult for the DM to get the priority weights of all the alternatives directly. But the DM can provide the relative weight of each pair alternatives with respect to a criterion more easily, and then construct a reciprocal comparison matrix in the AHP. Saaty [25] introduced the reciprocal comparison matrix as follows:

Definition 1 [25]. A reciprocal comparison matrix R is represented by

$$R = \begin{pmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{n1} & r_{n2} & \cdots & r_{nn} \end{pmatrix}$$

where r_{ij} indicates that the alternative x_i is r_{ij} times preferred than the alternative x_j under a criterion, and $r_{ij} = 1/r_{ji}, r_{ii} = 1, \quad i,j \in \{1, 2, \ldots, n\}$.