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The gradient evolution algorithm: A new metaheuristic

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ABSTRACT

This study presents a new metaheuristic method that is derived from the gradient-based search method. In an exact optimization method, the gradient is used to find extreme points, as well as the optimal point. This study modifies a gradient method, and creates a metaheuristic method that uses a gradient theorem as its basic updating rule. This new method, named gradient evolution, explores the search space using a set of vectors and includes three major operators: vector updating, jumping and refreshing. Vector updating is the main updating rule in gradient evolution. The search direction is determined using the Newton–Raphson method. Vector jumping and refreshing enable this method to avoid local optima. In order to evaluate the performance of the gradient evolution method, three different experiments are conducted, using fifteen test functions. The first experiment determines the influence of parameter settings on the result. It also determines the best parameter setting. There follows a comparison between the basic and improved metaheuristic methods. The experimental results show that gradient evolution performs better than, or as well as, other methods, such as particle swarm optimization, differential evolution, an artificial bee colony and continuous genetic algorithm, for most of the benchmark problems tested.

1. Introduction

Optimization problems arise in many different fields. Numerous studies focus especially on developing optimization methods. In general, optimization methods can be divided into two different categories: exact and non-exact methods. Exact methods ensure that the result obtained is the optimal solution; however, exact methods, such as the Lagrange and Simplex methods, require complex computation. Therefore, difficulties often arise in solving complex problems. Non-exact methods, such as numerical methods, heuristic and metaheuristic methods, entail easier calculation; even though they cannot ensure an optimal solution, most can reach a near-optimal solution. As the complexity of a problem increases, non-exact methods are more suitable than exact methods. Recently, there have been many studies on heuristic and metaheuristic methods. Most metaheuristic methods use evolution and swarm behaviors. Genetic algorithms (GA) [18], memetic algorithms (MA) [28], differential evolution (DE) algorithm [41] and bacteriological algorithms (BA) [5] are all inspired by cultural evolution. Ant colony optimization (ACO) algorithm [12], particle swarm optimization (PSO) algorithm [13], an artificial bee colony (ABC) algorithm [23], an intelligent water drop algorithm [38] and the bat algorithm (BA) [47] involve swarm intelligence methods.

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This study proposes a new metaheuristic that is derived from a basic optimization method, namely gradient-based method optimization. The gradient-based method has been widely applied in optimization problems. The gradient defines the curve of a function. A positive gradient represents an increasing function, while a negative gradient corresponds to a decreasing function. When the gradient is zero, the curve is flat. This point is called an extreme or stationary point. An optimal solution must be located at an extreme point. Gradient-based searching methods, such as the Newton method, the conjugate direction and the Quasi-Newton method rely on this concept [6]. The conjugate gradient, proposed by Hestenes and Stiefel [17], is a well-known method for solving problems in a linear equation system. Many optimization methods are based on this method. Ibtissem and Nouredine [19] combined the conjugate gradient with a differential evolution (DE) algorithm in solving a neural network problem. Herein, the conjugate gradient is used for a local search, while the DE algorithm is used for the global search. A similar concept was also used by Shahidi et al. [39] and Tingsong et al. [42], who combined the conjugate gradient with GA. Another method that integrates the gradient-based method with an evolutionary algorithm was proposed by Wu and Yu [44], who used a gradient descent mechanism to generate offspring for evolutionary programming. Chootinan and Chen [11] and Sardar et al. [37] used a gradient-based repair to repair an unfeasible solution. The gradient information is used to direct the unfeasible solutions towards the feasible region. These previous papers show the importance of the information derived from the gradient; however, these methods obtain the gradient by deriving the objective function. Unfortunately, some complex problems are non-differentiable functions. The proposed algorithm, named gradient evolution (GE) algorithm, is a new, population-based metaheuristic method. Since the main updating rule for the GE algorithm is derived from a gradient estimation method, the center differencing approach, this algorithm is applicable in many different optimization problems involving both differentiable and non-differentiable functions. In this study, the GE algorithm is first introduced and then evaluated using some basic test functions in an optimization field. These test functions are also used by most papers that propose a new optimization algorithm [4,23,30,49].

The remainder of this paper is organized as follows. Section 2 briefly reviews the basic theories involved in the GE algorithm. In Section 3, the GE algorithm is explained in detail. In order to validate the proposed method, a computational experiment is performed and discussed in Section 4. In this section, the proposed GE algorithm is tested using well-known benchmark optimization problems. The result is then compared with other metaheuristic methods. Finally, concluding remarks are given in Section 5.

2. Literature study

The population-based search algorithm proposed in this paper uses gradient theory, the Taylor series theorem and the Newton–Raphson Method. The following presents a brief overview of these theories.

2.1. The gradient

If a function $f:\mathbb{R}^n \to \mathbb{R}$ is differentiable at point x in $\mathbb{R}^n$, the derivative of $f(x)$ is described as the change in $f(x)$, as x changes [24]. For a real-value, single-variable function, the derivative can be visualized geometrically as the slope of the tangent or 0 gradient at point$x$. As shown in Fig. 1, the gradient of $f(x)$ at point x_0 can be approximated by drawing a secant line through $f(x_0)$ and $f(x_0+\Delta x)$. The gradient, denoted by m, is calculated as follows:

$$m = \frac{\Delta y}{\Delta x} = \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x} \tag{1}$$

As Δx becomes smaller, the point, $f(x_0+\Delta x)$, becomes closer to $f(x_0)$. This yields a more accurate approximation result. Therefore, as Δx approaches zero, the secant line becomes the tangent of $f(x)$ at the point, x_0. The derivative of $f(x)$, which is also the gradient at point x_0, is defined as follows:

$$m = \lim_{\Delta x \to 0} \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x} \tag{2}$$

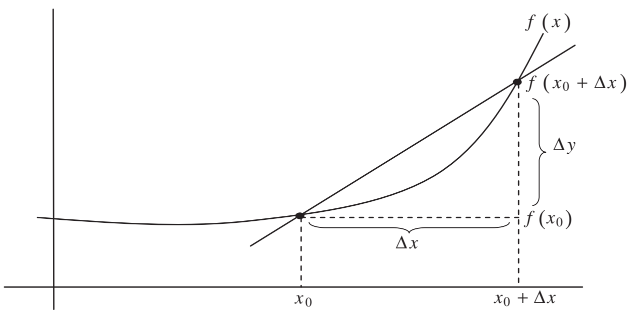

Fig. 1. Gradient approximation.