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Monotonic similarity measures between intuitionistic fuzzy sets and their relationship with entropy and inclusion measure

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ABSTRACT

Similarity measure is an important tool for determining the degree of similarity between two objects. Since the introduction of intuitionistic fuzzy sets, similarity measures between intuitionistic fuzzy sets have gained attention for their application in various fields. In this paper, the monotonicity of similarity measures between intuitionistic fuzzy sets are investigated. Firstly, by means of analyzing the geometrical relation between intuitionistic fuzzy sets, three types of monotonicity properties are defined. And then, the properties and the relationship with entropy and inclusion measure of these monotonic similarity measures are discussed. Finally, some existing monotonic similarity measures in literature are summarized and some new monotonic similarity measures and their applications are proposed and presented.

1. Introduction

Since the introduction of the theory of fuzzy sets [36], a number of approaches and theories treating imprecision and uncertainty have been proposed and studied. Intuitionistic fuzzy sets, proposed by Atanassov [1], are the higher order fuzzy sets which are capable of dealing with vagueness. Due to the properties of modelling uncertainty and the lack of information precision, intuitionistic fuzzy sets have been widely studied and applied in a variety of areas such as pattern recognition [6,20], logic programming [2,3], image segmentation [7,19], and medical diagnosis [9,37].

Similarity measure is an important tool for determining the degree of similarity between two objects. With the development of intuitionistic fuzzy sets, great efforts have been devoted to develop new similarity measures of intuitionistic fuzzy sets and study their applications. Li and Cheng [20] presented several similarity measures and applied them into pattern recognitions. Liang and Shi [24] proposed several similarity measures to overcome the weakness of the similarity measures given by Li and Cheng [20]. Mitchell [28] adopted a statistical approach and interpreted intuitionsitic fuzzy sets as ensembles of ordered fuzzy sets to modify Li and Cheng's similarity measures [20]. Hung and Yang [14] provided a similarity measure between intuitionistic fuzzy sets based on Hausdorff distance. Liu [26] proposed some similarity measures by means of the geometrical representation of an intuitionistic fuzzy set. Hung and Yang [15] presented several similarity measures based on L_p metric. Li et al. [22] summarized and compared some similarity measures by the counter-intuitive examples in pattern recognition. Ye [35] proposed a cosine similarity measure and a weighted cosine similarity measure. Hwang et al. [17] introduced a similarity measure induced by the Sugeno integral. Boran and Akay [6] proposed a biparametric similarity measure. Iancu [18] extended some crisp cardinality measures to measures for intuitionistic fuzzy sets and proposed several similarity

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measures based on Frank t-norms family. Xu [33] introduced several similarity measures of intuitionistic fuzzy sets by considering the weight of elements and applied them into multiple attribute decision making. Szmidt and Kacprzyk [31] proposed a similarity measure for intuitionistic fuzzy sets and applied it into supporting medical diagnostic reasoning. Xu and Yager [34] improved the similarity measure introduced by Szmidt and Kacprzyk [31] and applied it into consensus analysis in group decision making. Liang and Wei [25] proposed a similarity measure according to the relationship between entropy and similarity measure. Farhadinia [10] introduced a similarity measure on the basis of a distance defined on an interval by using convex combination of endpoints. Zhou et al. [38] presented an intuitionistic fuzzy ordered weighted cosine similarity measure by using the cosine similarity measure and the generalized ordered weighted averaging operator. Meng and Chen [27] proposed a construction approach to obtain the similarity measure by means of entropy and studied the application of entropy and similarity measure in pattern recognition. Ban [5] introduced some measures on intuitionistic fuzzy sets and discussed their applications. Atanassov et al. [4] introduced the similarity measures of intuitionistic fuzzy sets induced by distance and pseudo distances. Szmidt [32] introduced the axiomatic relation between similarity measures and distance measures and presented some counter-intuitive results given by the traditional similarity measures. Li et al. [23] investigated the relationship between similarity measure and entropy of intuitionistic fuzzy sets. Papakostas et al. [29] provided a detailed analysis of the distance and similarity measure for intuitionistic fuzzy sets from a pattern recognition point of view.

At present, there are several axiomatic definitions of similarity measure of intuitionistic fuzzy sets. To the best of our knowledge, the first axiomatic definition was introduced by Li and Cheng [20] in 2002. After that, Li et al. [22], Hung and Yang [15], Xu [33] and Li et al. [23] modified the definition given by Li and Cheng [20] and proposed several new axiomatic definitions of the similarity measure between intuitionistic fuzzy sets. In general, the similarity measures used in the literature are normally demanded to fulfill the following natural properties:

(p1) $0 \leqslant S(A,B) \leqslant 1$;
(p2) $S(A,A) = 1$;
(p3) $S(A,B) = S(B,A)$;
(p4) If $A \subseteq B \subseteq C$, then $S(A,C) \leqslant S(A,B)$ and $S(A,C) \leqslant S(B,C)$.

Where S is a similarity measure, and $A,\ B$ and C are intuitionistic fuzzy sets. The property (p2) indicates that S is reflexive; (p3) shows that S is symmetric; (p4) describes the intuition of a similarity measure which is regarded as monotonicity property.

In this paper we investigate the monotonicity property of the similarity measure between intuitionistic fuzzy sets. There are several reasons which motivate us to do this research. Firstly, the monotonicity property is obviously a key property to define a similarity measure. The property (p4), however is too weak. In fact, there are plenty of formulas which can satisfy the above natural properties, but not all of them can meet our intuitions. In literature, for example [22,10,32], we can find some counter-intuitive examples of some existing similarity measures. Secondly, sometimes we hope to compare similarities without considering what the similarity measure is. By the property (p4), however, we only can compare $S(A,C)$ with $S(A,B)$ or compare $S(A,C)$ with $S(B,C)$ when $A,\ B$ and C satisfy $A \subseteq B \subseteq C$. In order to address this issue, we need to investigate more monotonicity properties which meet our intuitions. Thirdly, we know that similarity measure, entropy [30] and inclusion measure [8,11,12] are three types of information measures of intuitionistic fuzzy sets and they can be transformed into each other. But only several specific transformation formulas can be found and there are no general formulas or functions in the previous literature [23,29]. The problem lies the condition $A \subseteq B \subseteq C$ of the property (p4) is too strong to be satisfied. Finally, as we all know, the methods for calculating similarity of fuzzy sets can be divided into three ways: distance-based methods, set-theoretic operations-based methods, and implication-based methods. For intuitionistic fuzzy sets, however, most of the methods for calculating similarities are distance-based. To generate more similarity measures between intuitionistic fuzzy sets, we also need to investigate the monotonicity of similarity measure.

The rest of this paper is organized as follows. In Section 2, we introduce some basic concepts and definitions which will be used. In Sections 3–5, we propose three types of monotonic similarity measures and investigate their properties and relationship with entropy and inclusion measure. In Section 6, we summarize some existing monotonic similarity measures, propose some new monotonic similarity measures, and present several applications of the proposed monotonic similarity measures. Finally, conclusions are made in Section 7.

2. Preliminaries

Throughout this paper, $\mathtt{IFSs}(X)$ is the class of all intuitionistic fuzzy sets of universal X; $\mathcal{F}(X)$ is the class of all fuzzy sets of universal X; $\mathcal{P}(X)$ is the class of all crisp sets of universal X; $\emptyset$ stands for the empty set, and the operation c denotes the complement operation.

Definition 2.1 (*Atanassov [1]*). An intuitionistic fuzzy sets A in $\mathtt{IFSs}(X)$ is defined as

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$$