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Aggregation of convex intuitionistic fuzzy sets

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ABSTRACT

Aggregation of intuitionistic fuzzy sets is studied from the point of view of preserving various kinds of convexity. We focus on aggregation functions for intuitionistic fuzzy sets. These functions correspond to simultaneous separate aggregations of the membership as well as of the nonmembership indicators. It is performed by means of the so-called representable functions. Sufficient and necessary conditions are analyzed in order to guarantee that the composition of two intuitionistic fuzzy sets preserves convexity.
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1. Introduction

Different kinds of convexity have been introduced in the literature, first to handle non-fuzzy sets and then for fuzzy sets and intuitionistic fuzzy sets. These definitions allow us to deal with any possible kind of non-fuzzy, fuzzy or intuitionistic fuzzy set. Therefore, suitable notions are introduced for cases when the universe is not necessarily a linear space. We work then with abstract convexities that come from [21].

We devote our study to this class of sets because convexity is one of the most important aspects in the study of geometric properties of not only standard sets, but also intuitionistic fuzzy sets, mainly arising –and indeed playing a crucial role– in applications connected to optimization and control (see [1,27]).

In particular we are interested in the study of the intersection of two convex IF-sets, but we will consider a more general framework, by using aggregation functions instead of the particular case of t-norms. Accordingly, the use of aggregation functions for intuitionistic fuzzy sets immediately carries the question of characterizing those aggregation functions that preserve convexity in some way. In the present paper we address this kind of questions.

The structure of the paper goes as follows: In Section 2, we recall the notions of a fuzzy set and an intuitionistic fuzzy set. In Section 3, we introduce the convex non-fuzzy and fuzzy sets and establish some equivalent definitions. Section 4 is devoted to the study of convexity for intuitionistic fuzzy sets. Finally, in Section 5, we analyze the behaviour of a generalization of the intersection of two convex intuitionistic fuzzy sets. A final section of concluding remarks and some open problems closes the paper.

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2. Basic concepts

In this section we carry out a brief introduction to fuzzy sets and intuitionistic fuzzy sets for a better understanding of the main body of the paper.

Let us start by recalling the standard definition of a fuzzy set.

Definition 1 ([29]). Let X be a nonempty set, usually called the *universe*. A *fuzzy set* A in X is defined by means of a map $\mu_A : X \to [0,1]$. The map μ_A is said to be the *membership function* (or *indicator*) of A.

The *support* of A is the non-fuzzy set $Supp(A) = \{t \in X : \mu_X(t) \neq 0\} \subseteq X$, whereas the *kernel* of A is the non-fuzzy set $Ker(A) = \{t \in X : \mu_X(t) = 1\} \subseteq X$. The fuzzy set A is said to be *normal* provided that it has nonempty kernel.

Given $\alpha \in (0,1]$, the non-fuzzy subset of X defined by $A_\alpha = \{t \in X : \mu_A(t) \geqslant \alpha\}$ is said to be the *α-cut (level set)* of the fuzzy set A.

Notice that through Definition 1, the fuzzy set A is indeed identified with its membership function μ_A. It is quite common to use both these notations, namely A and μ_A, interchangeably. A nice and alternative study of the concept of fuzzy set and its corresponding α-cuts was done in [5].

The object of our study is the intuitionistic fuzzy sets (IF-sets for short) introduced by Atanassov in [2,3] and in more details studied in [4]. Although they were seen at the beginning just as an extension of the ordinary fuzzy sets, quickly they were revealed to have essentially different features. A suitable example, that highlights this fact, appeared in [4].

Example 2. Let X be the set of all countries with elective governments. Assume that we know for every country $x \in X$ the percentage of the electorate that have voted for the corresponding government. Denote it by $M(x)$ and let $\mu(x) = M(x)/100$ (degree of membership, validity, etc.). Let $v(x) = 1 - \mu(x)$. This number corresponds to the part of electorate who have not voted for the government. By fuzzy set theory alone we cannot consider this value in more detail. However, if we define $v(x)$ (degree of non-membership, non-validity, etc.) as the ratio of votes given to parties or persons outside the government, then we can show the part of electorate who have not voted at all or who have given an invalid vote, and the corresponding ratio will be $\pi(x) = 1 - \mu(x) - v(x)$. Thus we can construct the set

$$\{(x, \mu(x), v(x)) | x \in X\}$$

and it is obvious that

$$0 \leqslant \mu(x) + v(x) \leq 1.$$

More precisely, the definition of an IF-set is given by:

Definition 3. Let X be a universe. A pair $A = (\mu_A, v_A)$ where μ_A and v_A are two functions from X to $[0,1]$ fulfilling that $\mu_A(x) + v_A(x) \leqslant 1$ for all $x \in X$ is called an *intuitionistic fuzzy set (IF-set)* in X.

The functions μ_A, v_A are its *membership* and *nonmembership* functions (or indicators, or degrees), respectively.

Also defined is function $\pi_A : X \to [0,1]$ through $\pi_A(x) = 1 - \mu_A(x) - v_A(x), \forall x \in X$ corresponding to the *degree of uncertainty or indeterminacy*.

Obviously, for every ordinary fuzzy set $\pi_A(x) = 0$ for each $x \in X$, and these sets have the form $\{(x, \mu(x), 1 - \mu(x)) | x \in X\}$. However, it is clear that IF-sets can be different from ordinary fuzzy sets.

In order to generalize the definitions of union, intersection, complementation and inclusion given by Atanassov [3], we will consider here the generalizations given by Deschrijver and Kerre presented in 2002 [13]. They are based on t-norms and t-conorms (see e.g. [19]), so we will start by recalling these concepts:

Definition 4. A map $T : [0,1] \times [0,1] \to [0,1]$ is said to be a *triangular norm* (*t-norm* for short) if it satisfies the following conditions:

- *Associativity*: $T(T(x,y),z) = T(x, T(y,z))$ for every $x, y, z \in [0,1]$.
- *Commutativity*: $T(x,y) = T(y,x)$ for every $x, y \in [0,1]$.
- *Monotonicity*: $T(y,x) \leqslant T(z,x)$ for all $x, y, z \in [0,1]$ with $y \leqslant z$.
- *Boundary conditions*: $T(x,1) = x$ for all $x \in [0,1]$.

Similarly, a map $S : [0,1] \times [0,1] \to [0,1]$ is said to be a *triangular conorm* (*t-conorm* for short) if it satisfies the conditions of associativity, commutativity, and monotonicity, as well as the following boundary conditions: $S(x,0) = x$ for every $x \in [0,1]$.

We may notice that if T is a t-norm, then the map $S : [0,1] \times [0,1] \to [0,1]$ given by $S_T(x,y) = 1 - T(1-x, 1-y)$ for every $x, y \in [0,1]$ is a t-conorm. T and S_T are said to be *dual* or *complementary*.

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