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Research on scheduling problems with general effects of deterioration and learning

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ABSTRACT

Jobs with deterioration and learning effect co-exist in many industry engineering and logistics management situations. In this note, the general effects of deterioration and learning are considered. It is proved that some single machine scheduling problems are still polynomially solvable under the proposed model. The note also shows that some results in Lee and Lai's recent paper (Lee and Lai, 2011) are incorrect by a counterexample.

1. Introduction

In classical scheduling it is assumed that the processing times of the jobs are constant. However, in many realistic settings, because firms and employees perform a task over and over again, they learn how to perform more efficiently. The production facility (a machine, a worker) improves continuously over time. As a result, the production time of a given product is shorter after it is scheduled, rather than the earlier process in the sequence. This phenomenon is known as the "learning effect" in the literature [1,5,10,12,14]. On the other hand, job deterioration appears, e.g., in scheduling maintenance jobs or cleaning assignments [6,8,9,11,13], where any delay in processing a job is penalized by incurring additional time for accomplishing the job. Extensive surveys of different scheduling models and problems involving jobs with start time dependent processing times can be found in Cheng et al. [3] and Gawiejnowicz [4]. An extensive survey of research related to scheduling with learning effects was provided by Biskup [2].

The recent paper "Scheduling problems with general effects of deterioration and learning" [7] addresses the single machine scheduling problems with general effects of deterioration and learning in which the actual job processing time is a general function on the processing times of the jobs already processed and its scheduled position. For the single-machine case, the authors showed that the scheduling problems to minimize makespan, total completion time and sum of the lth ($l > 0$) power of completion times can be solved in polynomial time. In addition, they also showed that the problems of minimizing the total weighted completion time, the maximum lateness, the maximum tardiness and the total tardiness are polynomially solvable under certain agreeable conditions. In this note, some counter-examples are given to show the

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incorrectness of the main results in Lee and Lai [7]. Besides, a revised model of Lee and Lai [7] is also provided. We show that some single machine scheduling problems are still polynomially solvable under the revised model.

We shall follow the notation and terminology given in Lee and Lai [7]. There are n jobs $J_1, J_2, \ldots, J_n$ given to be processed on a single-machine. Each job J_j has a due-date d_j and a weight w_j. All jobs are non-preemptive and are ready for processing at time 0. The machine can handle at most one job at a time and cannot stand idle until the last job assigned to it has finished processing. The actual processing time of job J_i $(i = 1, 2, \ldots, n)$ if scheduled in position r is

$$p^A_{j[r]} = p_j f\left(\sum_{k=1}^{r-1} \beta_k p_{[k]}, r\right), \quad r,\ j = 1, 2, \ldots, n, \tag{1}$$

where p_j is the normal processing time of job J_j, $p_{[k]}$ is the normal processing time of a job if it is scheduled in the kth position in a sequence, and $\beta_i > 0$ for all i. It is assumed that f is a general function satisfying the following conditions: (1) $f : [0, \infty) \times [1, \infty) \to (0, \infty)$ is a differentiable non-decreasing function with respect to the first variable x, non-increasing with respect to the second variable y; (2) $f_x(x, y_0) = \frac{\partial}{\partial x} f(x, y_0)$ is non-decreasing with respect to x for every fixed y_0 and $f(0, 1) = 1$. The first condition implies that the actual job processing times might be prolonged due to the deterioration effect which depends on the processing times of the jobs that are already processed. The second condition implies that the actual job processing times might be shortened due to the learning effect which depends on its scheduled position.

For a given schedule π, let $C_j = C_j(\pi)$ denote the completion time of job J_j. The objective functions to be minimized are makespan $C_{\max} = \max\{C_j | j = 1, 2, \ldots, n\}$, total completion time $\sum C_j$, total completion time and sum of the lth $(l > 0)$ power of completion times $\sum C_j^l$, total weighted completion time $\sum w_j C_j$, maximum lateness $L_{\max} = \max\{L_j | j = 1, 2, \ldots, n\}$, where $L_j = C_j - d_j$, maximum tardiness $T_{\max} = \max\{T_j | j = 1, 2, \ldots, n\}$, where $T_j = \max\{C_j - d_j, 0\}$, and total tardiness $\sum T_j$.

Lee and Lai [7] arrived at the following results in the single-machine scheduling problems:

Theorem 1′ (Theorem 1, [7]). *For the problem* $1|p^A_{j[r]} = p_j f\left(\sum_{k=1}^{r-1} \beta_k p_{[k]}, r\right)|C_{\max}$, *the optimal schedule is obtained by the shortest processing time (SPT) rule (i.e., by sequencing jobs in non-decreasing order of* p_j*).*

Theorem 2′ (Theorem 2, [7]). *For the problem* $1|p^A_{j[r]} = p_j f\left(\sum_{k=1}^{r-1} \beta_k p_{[k]}, r\right)|\sum w_i C_i$, *the optimal schedule is obtained by the weighted shortest processing time (WSPT) rule if the processing times and the weights are agreeable* $\left(\text{i.e., by sequencing jobs in non-decreasing order of } \frac{p_j}{w_j}\right)$.

Theorem 3′ (Theorem 3, [7]). *For the problem* $1|p^A_{j[r]} = p_j f\left(\sum_{k=1}^{r-1} \beta_k p_{[k]}, r\right)|L_{\max}$, *the optimal schedule is obtained by the earliest due date (EDD) rule if the job processing times and the due dates are agreeable (i.e., by sequencing jobs in non-decreasing order of* d_j*).*

Theorem 4′ (Theorem 4, [7]). *For the* $1|p^A_{j[r]} = p_j f\left(\sum_{k=1}^{r-1} \beta_k p_{[k]}, r\right)|\sum T_i$ *problem, the optimal schedule is obtained by the EDD rule if the job processing times and the due dates are agreeable.*

2. Counter-examples

In the following, we will show that Theorems 1′–4′ are not correct by a counter-example.

Counter-Example 1. Let $f\left(\sum_{k=1}^{r-1} \beta_k p_{[k]}, r\right) = \left(1 + \sum_{k=1}^{r-1} \beta_k p_{[k]}\right)^{\alpha}$, where $\alpha \geqslant 1$. $n = 3, p_1 = 1, p_2 = 2, p_3 = 200, w_1 = 3, w_2 = 2, w_3 = 1, d_1 = 4, d_2 = 10, d_3 = 200, \beta_1 = 2, \beta_2 = 3$, and $\alpha = 2$.

According to the result of Theorem 1′, we can obtain the schedule $[J_1, J_2, J_3]$, then we have

$$C_1 = 1, C_2 = 1 + 2 * (1 + 2 * 1)^2 = 19,$$
$$C_3 = 1 + 2 * (1 + 2 * 1)^2 + 200 * (1 + 2 * 1 + 3 * 2)^2 = 16219.$$

Hence, $C_{\max} = C_3 = 16219$.

However, if the schedule is $[J_2, J_1, J_3]$, then we have

$$C_2 = 2, C_1 = 2 + 1 * (1 + 2 * 2)^2 = 27,$$
$$C_3 = 2 + 1 * (1 + 2 * 2)^2 + 200 * (1 + 2 * 2 + 3 * 1)^2 = 12827.$$

Hence, $C_{\max} = C_3 = 12827$.