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A dual representation simulated annealing algorithm for the bandwidth minimization problem on graphs

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ABSTRACT

The bandwidth minimization problem on graphs (BMPG) consists of labeling the vertices of a graph with the integers from 1 to n (n is the number of vertices) such that the maximum absolute difference between labels of adjacent vertices is as small as possible. In this work we develop a DRSA (Dual Representation Simulated Annealing) algorithm to solve BMPG. The main novelty of DRSA is an internal dual representation of the problem used in conjunction with a neighborhood function composed of three perturbation operators. The evaluation function of DRSA is able to discriminate among solutions of equal bandwidth by taking into account all absolute differences between labels of adjacent vertices. For better performance, the parameters of DRSA and the probabilities for selecting the perturbation operators were tuned by extensive experimentation carried out using a full factorial design. The benchmark for the proposed algorithm consists of 113 instances of the Harwell-Boeing sparse matrix collection; the results of DRSA included 31 new upper bounds and the matching of 82 best-known solutions (22 solutions are optimal). We used Wilcoxon signed-rank test to compare best solutions produced by DRSA against best solutions produced by three state of the art methods: greedy randomized adaptive search procedure with path relinking, simulated annealing, and variable neighborhood search; according to the comparisons done, the quality of the solutions with DRSA is significantly better than that obtained with the other three algorithms.

1. Introduction

The bandwidth minimization problem on graphs (BMPG) was originated at the Jet Propulsion Laboratory in 1962 when Harper [16] studied the problem of labeling the vertices of a hypercube in n dimensions with the integers from 1 to 2^n in such a way that summation, over all neighboring pairs of vertices, of the absolute differences of integers assigned to vertices is minimal. Harary [15] proposed the problem independently of Harper.

Let $G = (V, E)$ be a finite undirected graph with $|V| = n$ and $|E| = m$. A labeling τ of G is a bijective mapping from the vertices of G to the set $\{1, 2, \ldots, n\}$. The bandwidth of G for τ, written $B_\tau(G)$, is $B_\tau(G) = \max\{|\tau(i) - \tau(j)| : (i,j) \in E\}$. BMPG is the problem of finding a labeling τ^* such that the bandwidth of G for τ^* is as small as possible: $B_{\tau^*}(G) = \min\{B_\tau(G)\}\ \forall\tau$.

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In the adjacency matrix $A = (a_{ij})$ of graph $G = (V, E)$ each non-zero element a_{ij} of A is in a diagonal located $|i - j|$ positions away from main diagonal. The distance from the farthest non-zero element to main diagonal is the bandwidth of the matrix; the term bandwidth is used because the non- zero elements are located in a band with respect to main diagonal. The bandwidth of a matrix A can be reduced by simultaneous permutations of rows and columns of the adjacency matrix; in fact simultaneous exchange of rows i and j, and columns i and j is equivalent to a label exchange of nodes i and j in the graph G.

Bandwidth minimization is desirable for a wide range of applications, for instance see [24,3,12,2]. The BMPG is NP-Complete [25], and a number of exact, greedy, and meta-heuristic algorithms have been developed to solve it. The most important exact algorithms are those developed by Del Corzo and Manzini [10]; Caprara and Salazar-González [5]; Martí et al. [20]; and Cygan and Pilipczuk [8,9]. In the group of greedy algorithms the best known are the Gibbs, Poole, and Stockmeyer (GPS) algorithm [13]; and the Cuthill–McKee (CM) algorithm [6]. In the class of meta-heuristic algorithms we found genetic programming [17], genetic algorithm [19], ant colony [18], tabu search [21], simulated annealing (SA) [11,27], greedy randomized adaptive search procedure with path relinking (GRASP-PR) [26], scatter search–tabu search [4], node-shift with hill climbing [19], and variable neighborhood search (VNS) [23].

In this work we propose a meta-heuristic algorithm called DRSA (Dual Representation Simulated Annealing) to solve BMPG. The novelty of this algorithm is the use of two internal representations of solutions for the problem; given a permutation $\tau = (\tau_1, \tau_2, \ldots, \tau_n)$ of the integers from 1 to n, where n is the number of vertices of the graph, the first representation considers element τ_i as the label assigned to vertex i, and the second representation interprets τ_i as the vertex to which label i is assigned. In conjunction with the two internal representations we use a neighborhood function composed of three perturbation operators; two of the three operators work over the first internal representation and the third operator works over the second one; the combined effect of the three operators is more powerful than the effect of each operator alone. The evaluation function of DRSA is a modified version of the evaluation function introduced in [27].

To change a solution of the BMPG, one of the three perturbation operators of the neighborhood function is applied. The operator to be used is selected according to certain precomputed probabilities determined using a full factorial tuning process that will be explained in Section 4. In the same tuning process values for parameters of the simulated annealing algorithm, specifically for initial temperature, cooling rate, and final length of the Markov chain are determined.

DRSA was tested with a well-known benchmark of 113 instances of Harwell-Boeing sparse matrix collection [22]; we improved 31 best-known solutions and matched 82 upper bounds (22 of them are optimal). According to Wilcoxon signed-rank test, DRSA outperforms the methods GRASP-PR [26], SA [27], and VNS [23], with respect to the quality of the solutions.

The remainder of this document is organized as follows: Section 2 presents a review of some of the most relevant methods for bandwidth minimization; Section 3 presents DRSA; Section 4 describes the process for tuning the parameters of DRSA using a full factorial design; Section 5 shows the results of applying DRSA to the 113 instances of the benchmark; and Section 6 is a summary of main contributions of this paper.

2. Relevant related work

In this section some relevant algorithms for solving BMPG are briefly described. The reviewed algorithms are grouped in three categories: exact, greedy, and meta-heuristic. In Table 1 we summarize significant information about algorithms including a short note about each one.

2.1. Exact algorithms

Del Corzo and Manzini [10] introduced two exact algorithms for solving BMPG. First one is the MB-ID algorithm (Minimum Bandwidth by Iterative Deepening), and second one is the MB-PS algorithm (Minimum Bandwidth by Perimeter Search).

MB-ID algorithm is based on the concept of upper partial ordering (UPO): for a graph G with vertex set $V = \{1, 2, \ldots, n\}$, a UPO of length $k < n$ is an injective function from $\{1, 2, \ldots, k\}$ to $\{1, 2, \ldots, n\}$. A UPO assigns the integers $1, 2, \ldots, k$ to k vertices of G, say $\{v_{j_1}, v_{j_2}, \ldots, v_{j_k}\}$. MB-ID searches for a b-bandwidth solution, where b is initially a lower bound of the bandwidth, i.e., $b \leqslant B_{\tau^*}(G)$. If this search fails, the algorithm searches for a $(b+1)$-bandwidth solution and so on. In each search the algorithm begins with a UPO of length zero, and by means of a depth first search, vertices are added to the current UPO.

Caprara and Salazar-González [5] proposed an enumerating scheme for computing $B_{\tau^*}(G)$ based on the construction of partial layouts, where a partial layout of a subset of vertices $S \subset V$ is a function $\tau : S \to \{1, 2, \ldots, n\}$ such that $\tau_u \neq \tau_v$ for $u, v \in S$ and $u \neq v$. The basic idea is determining the minimum reachable bandwidth knowing the current partial layout. Based on which labels are already assigned, the algorithm estimates the minimum bandwidth that can be reached.

Martí et al. [20] introduced another exact algorithm for solving BMPG. This algorithm uses a greedy randomized adaptive search procedure (GRASP) [26] for determining initial upper bound of the bandwidth. Suppose GRASP finds a bandwidth of size k; then the exact algorithm begins by searching a bandwidth of size $k-1$. If a solution is found the algorithm searches for a bandwidth of size $k-2$, and so on. When the algorithm does not find a solution having a bandwidth of size $k-i$ it terminates and reports the optimal bandwidth of size $k-i+1$. The initial solution provided by the GRASP algorithm allows for the reduction of the search space that is explored.