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Generalized one-sided concept lattices with attribute preferences

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ABSTRACT

The main aim of this paper is to introduce the preference relations on generalized one-sided concept lattices, which represent a fuzzy generalization of FCA with classical object clusters and fuzzy attributes. In our case a preference relation is modeled by a linear well quasi-order on the set of all attributes. We describe concept forming operators based on a Galois connection, which is defined between the power set of objects and the fuzzy sets of attributes with lexicographic order induced by the preference relation. The representation theorem for such kind of concept lattices is also presented.

1. Introduction

Formal Concept Analysis (FCA [14]) represents a theory of data analysis for identification of conceptual structures among data sets. The mathematical theory of FCA is based on the notion of a formal context represented by a binary relation between a set of objects and a set of attributes. Consequently, a formal context is transformed into a complete lattice (called concept lattice) consisting of all conceptual abstractions (concepts) combining subsets of objects with subsets of shared attributes. As an effective tool for data analysis, FCA has been extensively applied to fields such as decision making, information retrieval, software engineering and knowledge discovery.

Classical FCA is suitable for a binary case. However, in practice there are natural examples where a many-valued or fuzzy description of object-attribute model is more convenient. Therefore, handling uncertainty, imprecise data or incomplete information has become an important research topic in the recent years in the field of FCA. We mention the approach of Bělohlávek [3–5] based on the logical framework of complete residuated lattices, the generalized concept lattices defined by Krajči [20], the work on multi-adjoint concept lattices [24,26,25,27] and other works related to FCA cf., [1,9,10,16–18,21–23,29–32].

A special case of fuzzy FCA is a one-sided concept lattice, where usually object clusters are considered as crisp subsets and attributes obtain fuzzy values, cf. [2,19] or [15]. In this case, an interpretation of resulting concepts is straightforward as in the classical FCA, instead of other fuzzy approaches with fuzzy subsets of objects, where interpretability often becomes

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problematic. From this point of view, one-sided concept lattices are practically applicable in the same way as the classical concept lattices.

One of the emerging problems in the usage of concept lattices in software engineering is the reduction of the number of concepts. One of the possible ways can be represented by involving another relevant information to the process of forming concept lattices. This can be given by preference relations, which are connected to rational decisions. Let us note that models of preferences are one of the essential components in decision making. For expressing preferences, the fundamental notions are relations, particularly (quasi)-orderings. Traditionally, a (weak) preference relation on a nonempty choice set X is defined as a complete, reflexive and transitive binary relation (a complete quasi-order) on X. This notion coincides with that of weak order which is used for the ranking of linearly ordered properties of objects. Its fuzzy variant has been quite well developed, for details we refer the reader to comprehensive papers [6,7].

Recently, the approach of preference modeling in the classical FCA was described by Obiedkov in [28]. Our main aim is to present a possible way how to introduce a preference relation defined on an attribute set into the process of creating one-sided concept lattices. For this purpose we use the notion of lexicographic product of complete lattices, which will provide an algebraic support for an evaluation of attributes. This represents a different approach for modeling preferences as that described in [28].

To illustrate our motivation, consider that the set of four cars $\{c_1, c_2, c_3, c_4\}$ is given. Assume that these cars are characterized by their fuel consumption, presence of two safety elements, namely ABS and ESP, and their total luggage space. The specific characteristics of these cars are given in Table 1.

Now, we would like to make some decision based on these properties. One possibility is to consider some "threshold" configuration and select all cars satisfying it. An example of such threshold can be: fuel consumption at most 7.0 l/100 km, presence of ABS and luggage space greater than 390 dm^3. As one can see, only c_1 and c_3 satisfy this configuration. Hence, considering such threshold configuration can effectively reduce the so-called decision space. Such simultaneous reduction is provided by the generalized one-sided concept lattice approach. In this case, the intent of a concept represents some maximal threshold which is satisfied by all objects in the extent set and the extent consists of all objects which satisfy the intent threshold. However, even after the reduction of a decision space, one need not be able to choose the best alternative: in our example the cars c_1 and c_3 have the first two properties common, but they differ in the remaining two ones. The car c_1 has the greater luggage space but ESP is missing. On the other hand, the car c_3 has ESP but the luggage space is smaller. Hence, if these two attributes are equally important (or indifferent), one cannot decide which car is better. This is caused by "pointwise" ordering of attribute values which is considered in the underlying direct product structure for evaluating attribute configurations.

However, in practice some attributes can be more important than others. Hence, one can assume that a preference relation on the set of attributes is given. A comparison of configurations can be done in such a way that the more preferred attributes dominate the less preferred ones, i.e., a configuration with a greater value of more preferred attribute is greater (better) than a configuration with a smaller value of the same attribute, regardless on the values of less preferred attributes. This corresponds to a lexicographic ordering of the underlying structure for attribute values evaluation. In our example, consider that fuel consumption is indifferent with the presence of ABS, these two attributes are more preferred than the presence of ESP, what is more preferred than the total luggage space. In this case the car c_3 will be better than the car c_1 and one should expect that this fact also appears in the resulting concept lattice.

As it was mentioned, the main aim of this paper is to describe the theory of generalized one-sided concept lattices with a preference relation on a set of attributes, based on a lexicographic order of attribute values.

The paper is organized as follows: we begin with an overview of the Galois connections and fuzzy concept lattices derived from these kind of mappings and we provide the basic definition of the so-called generalized one-sided concept lattices. Next, we recall the definition of the lexicographic product and we give some basic results concerning this notion. In Section 3 we describe so-called generalized one-sided concept lattices with a preference relation on the set of attributes. Further we involve concept forming operators based on the preference relation and derive their basic properties. An illustrative example of such operators is also presented. The last section contains the proof of the representation theorem for generalized one-sided concept lattices with a preference relation.

2. Preliminaries

In this section we describe the basic notions needed for our purposes. We start with the definition of (antitone) Galois connection.

Table 1
Cars with considered attribute characterizations.

	Fuel consumption	ABS	ESP	Luggage space
c_1	6.7 l/100 km	×		450 dm^3
c_2	7.3 l/100 km		×	550 dm^3
c_3	6.7 l/100 km	×	×	400 dm^3
c_4	5.5 l/100 km		×	350 dm^3