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The approximation set of a vague set in rough approximation space

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ABSTRACT

Vague set is a further generalization of fuzzy set. In rough set theory, a target concept may be a defined set, fuzzy set or vague set. That the target concept is a defined set or fuzzy set was analyzed in detail in our other papers respectively. In general, we can only get two boundaries of an uncertain concept when we use rough set to deal with the uncertain problems and can not get a useable approximation defined set which is a union set with many granules in Pawlak's approximation space. In order to overcome above shortcoming, we mainly discuss the approximation set of a vague set in Pawlak's approximation space in the paper. Firstly, many preliminary concepts or definitions related to the vague set and the rough set are reviewed briefly. And then, many new definitions, such as 0.5-crisp set, step-vague set and average-step-vague set, are defined one by one. The Euclidean similarity degrees between a vague set and its 0.5-crisp set, step-vague set and average-step-vague set are analyzed in detail respectively. And then, the conclusion that the Euclidean similarity degree between a vague set and its 0.5-crisp set is better than the Euclidean similarity degree between the vague set and the other defined set in the approximation space (U,R) is drawn. Afterward, it is proved that average-step-vague set is an optimal step-vague set because the Euclidean similarity degree between a vague set and its average-step-vague set in the approximation space (U,R) can reach the maximum value. Finally, the change rules of the Euclidean similarity degree with the different knowledge granularities are discussed, and these rules are in accord with human cognitive mechanism in a multi-granularity knowledge space.

1. Introduction

Since the fuzzy set theory was proposed by Zadeh in 1965 [46], it has been successfully applied to process many uncertain problems, such as fuzzy image segmentation, fuzzy clustering analysis, fuzzy control and so on. The uncertainty of an uncertain concept is described by a membership degree in the fuzzy set theory. In other words, a fuzzy set interprets the uncertain concepts with some membership function as well as a fundamental tool for revealing and analyzing uncertain problems, and it has been frequently used in real-life world applications. In fact, the fuzzy set theory is an extension of the classical Canter's set theory. A fuzzy set A is a class of objects which satisfies a certain property and each object x has

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a membership degree of A, denoted as $\mu_A(x)$. The membership function has the following characteristics: the single-value membership degree contains the evidences for both supporting and opposing x, and the value $\mu_A(x)$ almost is a subjective value which is probably different due to the different researchers.

Atanassov [2] proposed the intuitionistic fuzzy set as a generalization of the fuzzy set in 1986, and Gau and Buehrer [10] put forward the concept of the vague set in 1993. Afterward, Bustince and Burillo [5] drew a conclusion that the essence of the vague set was the same as that of the intuitionistic fuzzy set in 1996. Lu and Ng [14] analyzed the relationship between the vague set and the intuitionistic fuzzy set in detail, and they pointed out that the vague set was more natural than the intuitionistic fuzzy set to characterize fuzzy objects and it was not isomorphic to the intuitionistic fuzzy set. So the terms of both the vague set and the intuitionistic fuzzy set exist in different literatures or research papers. In this paper, the intuitionistic fuzzy set and the vague set are uniformly called the vague set due to the fact that many references are related to the vague set.

Gau and Buehrer [10] pointed out that the drawback of the single membership value in the fuzzy set theory is that the evidence for $x_i \in U$ and the evidence against $x_i \in U$ are in fact mined together. They also stated that the single value revealed nothing about its accuracy. To tackle this problem, Gau and Buehrer [10] proposed the notion of vague set, which allowed to use interval-based membership instead of using point-based membership as the fuzzy set. They employed a truth-membership function $t_V(x_i)$ and a false-membership $f_V(x_i)$ to characterize the lower bounds on $\mu_V(x_i)$. These bounds are used to create a subinterval on $[0,1]$, namely, $t_V(x_i) \leqslant \mu_V(x_i) \leqslant 1 - f_V(x_i)$. The vague set as well as the fuzzy set and the rough set [18], is not only an important tool to process uncertain problems [41] but also a typical soft computing method in data mining, especially, in the dynamic data mining field, knowledge has a greater uncertainty in the process of knowledge acquisition [52].

Vagueness, as well as fuzziness and roughness, has attracted the attentions of many researchers since the vague concept was proposed. And vagueness is current subject of vigorous debate in the philosophy of logic and language. Vague terms, such as 'tall', 'red', 'bald' and 'tadpole' have borderline cases (arguably, someone may be neither tall nor not tall), and they lack well-defined extensions (there is no sharp boundary between tall people and the rest). The phenomenon of vagueness poses a fundamental challenge to classical logic and semantics, which assumes that propositions are either true or false and that extensions are determinate [11,12,16,28]. Ronzitti [29] analyzed the vagueness and metaphysics which covers important questions concerning vagueness that arise in connection with the deployment of certain key metaphysical notions. Based on rough set theory, there are many papers discussing the relationships between the rough set and the vague set. Rough set theory seems to be well suited as a mathematical model of vagueness and uncertainty, and vagueness is a property of set (concept) and is strictly related to the existence of the boundary region of a set, whereas uncertainty is a property of elements of the set and is related to rough membership function [19,20]. Orowska [17] gave a formal framework to what was considered to be different from rough set theory ways of making vague concepts precise and described semantics in this framework. Skowron, Bazan and Swiniarski [3,26,27,30–37] proposed a rough set approach to search the approximate concept (or set) of vague concept within the adaptive learning framework, and the boundary regions of approximate concepts within the adaptive learning framework are satisfying the higher order vagueness condition, i.e., the boundary regions of vague concepts are not defined. Vagueness has been equated with the idea that, for several different reasons, an object can not always be considered as satisfying either A or not A as prescribed by classical logic [8].

The uncertainty of a vague concept has been an important issue since the vague set was proposed, and the fuzziness, roughness and vagueness are three main uncertainty metrics of the vague concept. In the Pawlak's approximation space, the uncertainty of vague concept has been studied, and the change rules of uncertainty have been analyzed in a multi-granularity knowledge space [3,17,20,31,35]. There are three types of knowledge that can be specified according to the rough set theory, and three corresponding types of algebraic structures appear in rough set theory [39]. Wolski [40] thought that rough set theory can be viewed as a modified semantics, and in order to be more precise, rough set provided a modal version of this semantics. Then Bonikowski [4] proposed a new formal approach to vagueness, and many important conditions concerning the membership relation for vague sets, in connection to multi-sets and fuzzy sets, were established as well. Granular computing is an umbrella term to cover any theories, methodologies, techniques, and tools that make use of information granules in complex problem solving [24,44]. And it should focus on formalizing information granules and unifying them to create a coherent methodological and developmental environment for intelligent system design and analysis, and the granular fuzzy decision support systems were built successfully [1,21,23,25]. Multi-granularity knowledge discovery and management has attracted a great deal of researchers attention, and a knowledge management and semantic modeling based on information granularity is proposed and they capture semantics, facilitate comprehension, and support communication mechanisms [22]. The uncertainty of the decision information systems is an important parameter for obtaining a good decision making.

Vagueness mainly reflects the boundary's uncertainty of a vague concept, and it has been studied by many researchers. However, it is very difficult to describe a vague concept with an approximation defined set in Pawlak's knowledge space. The vague set, as well as the rough set, focuses on finding the boundaries of a vague concept and neglects how to construct a defined set with many granules in current knowledge space as an approximation concept of the vague concept. Fuzzy set is a very successful approach to vagueness. In this approach, sets are defined by partial membership in contrast to defined membership used in the classical definition of a set. Rough set expresses vagueness not by means of membership but by employing the boundary region of a set. If the boundary region of a set is empty, it means that a particular set is defined, otherwise the set is rough. The non-empty boundary region of the set means that our knowledge about the set is not sufficient to define the set precisely [17,20,31]. In real-life world, any vague concept defined by philosophers [11,12] has the boundary region which is not definable by using fuzzy set or rough set. And this problem cannot be eliminated by 'static'