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Fault-tolerant Hamiltonian laceability of balanced hypercubes ☆

Qingguo Zhou [a], Dan Chen [a], Huazhong Lü [b,*]

[a] School of Information Science and Engineering, Lanzhou University, Lanzhou, Gansu 730000, PR China
[b] School of Mathematics and Statistics, Lanzhou University, Lanzhou, Gansu 730000, PR China

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ABSTRACT

The balanced hypercube, as a new variant of hypercube, has many desirable properties such as strong connectivity, high regularity and symmetry. The particular property of the balanced hypercube is that each processor has a backup processor sharing the same neighborhood. A Hamiltonian bipartite graph $G = (V_0 \cup V_1, E)$ is said to be Hamiltonian laceable if there is a Hamiltonian path between any two vertices $x \in V_0$ and $y \in V_1$. It has been proved that the balanced hypercube BH_n is Hamiltonian laceable for all $n \geqslant 1$. In this paper, we have proved that after at most $2n - 2$ faulty edges occur, BH_n remains Hamiltonian laceable for all $n \geqslant 2$, this result is optimal with respect to the number of faulty edges can be tolerated in BH_n.

1. Introduction

Design of interconnection networks is an essential part of large parallel processing and distributed systems. The topology of an interconnection network is usually represented by a graph. The hypercube network is one of the most popular and efficient interconnection networks, which possesses many excellent properties such as recursive structure, regularity, symmetry and small diameter. Though the hypercube is suited to both special-purpose and general-purpose, there is no network meeting all aspects of the requirements. Thus, many variants of the hypercube have been proposed, such as [3–6,17,22]. The balanced hypercube is one of those variants and its novel property is: each vertex of it has a backup (matching) vertex that shares the same neighborhood. With this special property, tasks running on a faulty vertex can be transferred to its backup vertex. In addition, odd-dimensional balanced hypercube has smaller diameter than that of standard hypercube with the same vertices [22]. With so many attractive properties above, the balanced hypercube was extensively investigated in the literature [10,15,16,23–27].

Paths are fundamental networks for parallel and distributed computation, and many simple and efficient path algorithms for solving various algebra and graph problems can be found in [12]. Also, paths can be used as control/data structures for distributed computation in networks [18]. Moreover, an application of Hamiltonian paths to a practical problem was encountered in the on-line optimization of a complex Flexible Manufacturing System [2].

A graph G is *Hamiltonian connected* if there exists a Hamiltonian path joining any two vertices of G. Any bipartite graph with more than two vertices is not Hamiltonian connected. Simmons [19] introduced the concept of Hamiltonian laceable for bipartite graphs: a bipartite graph $G = (V_0 \cup V_1, E)$ is *Hamiltonian laceable* if there is a Hamiltonian path between any two vertices x and y in different partite sets. Hamiltonian laceability of many famous network were studied, such as the

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* Corresponding author.
E-mail address: lvhz08@lzu.edu.cn (H. Lü).

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hypercube [7], star graphs [8], the folded hypercube [9], n-dimensional rectangular lattices [19] and the balanced hypercube [23]. Moreover, hyper-Hamiltonian laceability of the hypercube and the balanced hypercube were investigated in [16,13].

It is important to consider fault-tolerance since failure may occur when a large network is put into use. In [16], Lü et al. proved that the balanced hypercube is hyper-Hamiltonian laceability, that is, after exact one faulty vertex occur, say $u \in V_0$, there exists a Hamiltonian path of the faulty balanced hypercube between any two vertices in V_1, where V_0 and V_1 is bipartition of the balanced hypercube. So it is meaningful to consider Hamiltonian paths embedding problem of the balanced hypercube with only faulty edges. On the other hand, the balanced hypercube possesses some attractive advantages such that the hypercube dose not have, it is important to study whether the balanced hypercube have such property. To measure whether the network with at most k faulty edges has a Hamiltonian cycle, Hsieh et al. [8] proposed the concept k edge fault-tolerant Hamiltonian. Later Tsai et al. [20] extended this concept to Hamiltonian laceable as follows. A Hamiltonian laceable graph G is *k edge fault-tolerant Hamiltonian laceable* if $G - F$ remains Hamiltonian laceable for every $F \subset E(G)$ with $|F| \leqslant k$. The *edge fault-tolerant Hamiltonian laceability*, denoted by $\mathcal{H}_e^L(G)$, is the maximum integer k such that G is k edge fault-tolerant Hamiltonian laceable. In [11], Latifi et al. proved that after at most $n-2$ faulty edges occur, the hypercube keeps Hamiltonian laceable. Araki and Kikuchi [1] and Li et al. [14] show that bubble-sort graphs and Cayley graphs generated by transposition trees can tolerate $n-3$ faulty edges remaining Hamiltonian laceable, respectively.

The rest of this paper is organized as follows. In Section 2, we give a brief summary of concepts and some well-known properties of the balanced hypercube. In Section 3, we present our main results of this paper. Finally, we conclude this paper in Section 4.

2. Preliminaries

Let $G = (V, E)$ be a simple undirected graph, where V is the vertex-set of G and E is the edge-set of G. For a vertex x in G, the set of vertices adjacent to x is denoted by $N_G(x)$, also abbreviated for $N(x)$ when the context is clear. $|N_G(x)|$ is said to be the *degree* of x in G, denoted by $d(x)$. A *path* P from v_0 to v_n is a sequence of distinct vertices $v_0v_1\cdots v_n$ such that every pair of consecutive vertices are adjacent. We also denote the path $P = v_0v_1\cdots v_n$ by $\langle v_0, P, v_n\rangle$. The *length* of a path P is the number of edges in P, denoted by $l(P)$. For graph terminologies and notations not defined here, we follow [21].

Wu and Huang [22] gave the definition of BH_n as follows:

Definition 1. An n-dimensional balanced hypercube BH_n consists of 4^n vertices, each labeled by an n-bit string $(a_0, \ldots, a_{i-1}, a_i, a_{i+1}, \ldots, a_{n-1})$, where $a_i \in \{0,1,2,3\}(0 \leqslant i \leqslant n-1)$. An arbitrary vertex $(a_0, \ldots, a_{i-1}, a_i, a_{i+1}, \ldots, a_{n-1})$ in BH_n has the following $2n$ neighbors:

(1) $((a_0+1) \bmod 4, a_1, \ldots, a_{i-1}, a_i, a_{i+1}, \ldots, a_{n-1})$,
$((a_0-1) \bmod 4, a_1, \ldots, a_{i-1}, a_i, a_{i+1}, \ldots, a_{n-1})$, and
(2) $((a_0+1) \bmod 4, a_1, \ldots, a_{i-1}, (a_i + (-1)^{a_0}) \bmod 4, a_{i+1}, \ldots, a_{n-1})$,
$((a_0-1) \bmod 4, a_1, \ldots, a_{i-1}, (a_i + (-1)^{a_0}) \bmod 4, a_{i+1}, \ldots, a_{n-1})$.

In BH_n, the first coordinate a_0 of vertex $(a_0, \ldots, a_i, \ldots, a_{n-1})$ is called *inner index*, and other coordinates $a_i (1 \leqslant i \leqslant n-1)$ *i-dimension index*. Clearly, each vertex in BH_n has n pairs of neighbors differing in only inner index.

In addition, BH_n can be constructed from four copies of BH_{n-1} by adding a new dimension as the $(n-1)$-dimension index of every vertex in BH_n. Wu and Huang [22] gave the recursive definition of BH_n as follows:

Definition 2.

(1) BH_1 is a 4-cycle, whose vertices are labeled clockwise as $0, 1, 2, 3$, respectively.
(2) BH_{k+1} is constructed from 4 BH_ks. These four BH_ks are labeled $BH_k^{(0)}, BH_k^{(1)}, BH_k^{(2)}, BH_k^{(3)}$, where each vertex in $BH_k^{(i)} (0 \leqslant i \leq 3)$ has i attached as the new k-dimension index. Every vertex $v = (a_0, a_1, \ldots, a_{k-1}, i)$ in $BH_k^{(i)}$ $(0 \leqslant i \leq 3)$ has two extra neighbors:
   (a) $((a_0+1) \bmod 4, a_1, \ldots, a_{k-1}, (i+1) \bmod 4)$ and $((a_0-1) \bmod 4, a_1, \ldots, a_{k-1}, (i+1) \bmod 4)$, which are in $BH_k^{(i+1)}$ if a_0 is even.
   (b) $((a_0+1) \bmod 4, a_1, \ldots, a_{k-1}, (i-1) \bmod 4)$ and $((a_0-1) \bmod 4, a_1, \ldots, a_{k-1}, (i-1) \bmod 4)$, which are in $BH_k^{(i-1)}$ if a_0 is odd.

BH_1 and BH_2 are shown in Figs. 1 and 2, respectively.

Some basic properties of the balanced hypercube are given as follows:

Proposition 1 [22]. *The balanced hypercube is bipartite.*

Proposition 2 ([22,27]). *The balanced hypercube is vertex-transitive and edge-transitive.*