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Deriving three-way decisions from intuitionistic fuzzy decision-theoretic rough sets

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ABSTRACT

Three-way decisions with decision-theoretic rough sets (DTRSs) provide a new methodology to confront risk decision problems. The risk is associated with the loss function of DTRSs. Under the intuitionistic fuzzy environment, we combine the loss functions of DTRSs with intuitionistic fuzzy sets (IFSs). Considering the new evaluation format of loss function with intuitionistic fuzzy numbers (IFNs), we propose a naive model of intuitionistic fuzzy decision-theoretic rough sets (IFDTRSs) and elaborate its relevant properties in advance. At this point, a critical issue is the determination of three-way decisions. In the frame of IFDTRSs, we then explore deriving three-way decisions for single-period decision making. Based on the positive and negative characteristics of IFNs, we design three strategies to address IFNs and derive corresponding three-way decisions. Meanwhile, we compare the three strategies and summarize their own applicabilities. In order to accommodate multi-period scenarios, we further extend IFDTRSs to the multi-period situation. With the aid of the results of the single period decision making, we analyze three aggregation operations of IFDTRSs for multi-period information, which are DIFWA, DIFPA and DIFOA, respectively. By comparing these operations, an algorithm for deriving three-way decisions in multi-period decision making is designed. These results help us to make a reasonable decision in the intuitionistic fuzzy environment. Finally, an example is presented to elaborate on three-way decisions with IFDTRSs.

1. Introduction

Three-way decisions, consisting of acceptance, non-commitment (or further investigation) and rejection, are commonly encountered in problem solving strategies [57], such as investment decision [21,25], information filtering [14], text classification [16], risk decision [17,21], cluster analysis [23,59], government decision [28], web-based support systems [50], etc. The notion of three-way decisions is initially developed in the framework of rough sets [10,33]. More specifically, considering three pairwise disjoint regions of rough sets (i.e., positive region $\mathrm{POS}(C)$, boundary region $\mathrm{BND}(C)$ and negative region $\mathrm{NEG}(C)$), Yao [53–55] proposed three-way decisions with the aid of the risk decision semantics. The three-way decisions comprise positive rules, boundary rules and negative rules [53,54,56,57]. The positive rules associated with the positive region produce acceptance decisions. The negative rules coming with the negative region give rise to rejection decisions, while the boundary rules (coming with the boundary region) result in decisions of non-commitment. In connection with

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the three basic elements of three-way decisions, Hu [10] systematically discussed three-way decisions space and three-way decisions. As an extended model of rough sets, decision-theoretic rough sets (DTRSs) vastly push the development of three-way decisions [31,37,51,56,57]. In light of Bayesian decision procedure, DTRSs were proposed by Yao et al. [51,52]. The model involves practical decision semantics and considers relevant risks. Three-way decisions with DTRSs are derived from the minimum of the overall risk.

For three-way decisions with DTRSs, Liu et al. [30] summarized the existing literatures and classified these researches in detail. Azam and Yao [2] investigated the use of game-theoretic rough set (GTRS) model in exploring the relationship between the changes of probabilistic thresholds and their impacts on uncertainty levels of different regions. Deng and Yao [7] presented an information-theoretic approach to the interpretation and determination of probabilistic thresholds. Considering the costs of misclassification, Greco et al. [9] proposed a new dominance-based rough set approach based on DTRSs. In the viewpoint of the optimization, Jia et al. [12,13] discussed the determination of the thresholds and the reduction technology for DTRSs. In light of different attitudes of decision makers, Li and Zhou [17] discussed the relationships of loss functions and proposed a multi-view DTRS decision model. Considering both the misclassification cost and test cost, Li et al. [18] designed an algorithm for attribute reduction with the minimum total cost. In the context of DTRSs, Li et al. [19] verified its non-monotonic feature of attribute reduction and designed a corresponding algorithm. Based on the DTRS model, Lingras et al. [23] redefined a loss function and proposed a cluster quality index for rough clusters. With respect to an investment decision-making, Liu et al. [25] proposed a profit-based three-way approach with DTRSs. By analyzing the different combinations of loss functions, Liu et al. [26] proposed four-level probabilistic rules choosing criteria. In addition, Liu et al. [26] also listed some methods for estimating the loss function. Min and Zhu [32] studied the attribute reduction of data with error ranges and test costs. Through combining DTRSs and granular structures, Qian et al. [36] developed a multigranulation decision-theoretic rough set. With respect to multiple sets of decision preferences, Yang and Yao [49] discussed some aggregations of loss functions in a multi-agent DTRS model. Yao [53] employed relative values between loss functions to express the thresholds and reduced the estimation amount. In light of the loss function of DTRSs, Yu et al. [59] constructed a new clustering validity evaluation function. Zhang and Miao [61] established a fundamental reduction framework for two-category DTRSs. Under a multi-class environment, Zhou [62] provided a new formulation of multi-class DTRSs. With the aforementioned literatures, the determination of loss function of DTRSs is a pivotal issue. Nowadays, some uncertain evaluation scenarios enrich the determination of loss function and make it adapt to a certain decision making environment. For example, Liang et al. [20], Liang and Liu [21,22] and Liu et al. [27] effectively estimated the loss functions in the format of triangular fuzzy numbers, hesitant fuzzy sets, intervals, and typical stochastic functions. These works extend the range of applications of three-way decision with DTRSs.

Inspired by the researches reported in [20–22,27], intuitionistic fuzzy sets (IFSs) proposed by Atanassov [1], are also a significant evaluation format to describe the uncertainty. Unlike an ordinary fuzzy set [8,34,60], the concept of IFSs is a more flexible way to capture the uncertainty and has a duality property, which is characterized both by a membership degree and by a non-membership degree [3]. IFSs have been witnessed in numerous situations and mainly utilize intuitionistic fuzzy numbers (IFNs) to support the decisions [3–5,11,15,24,38–42,44–48]. For instance, Boran and Akay [3] proposed a new general type of similarity measure of IFSs and applied it to pattern recognition. Chaira and Ray [4] used IFSs for solving image edge detection. Based on the degrees of membership and non-membership of IFSs, Chen [5] elucidated the positive and negative evaluation effects (or optimism and pessimism) and developed a unipolar bivariate approach. Considering a combination of multigranulation rough sets with intuitionistic fuzzy rough sets, Huang et al. [11] developed a new multigranulation rough set model. Vlachos and Sergiadis [38] presented an information-theoretic approach to discrimination measures for IFNs and wielded it in the context of pattern recognition. In the field of multi-criteria decision making, Li [15], Liu and Wang [24], Xu [40] and Yager [47] discussed some aggregation operations of IFNs with different contexts. Unlike the existing works presented in [20–22,27], this paper introduces the IFN into DTRSs and proposes a new model of intuitionistic fuzzy decision-theoretic rough sets (IFDTRSs). By utilizing the positive and negative characteristic of IFNs [5], we design three strategies to address IFNs and deduce appropriate three-way decisions. In addition, there also exist some multi-period scenarios in real decision making procedure, e.g., multi-period (multi-stage) investment decision making, medical diagnosis, personnel dynamic examination, and military system efficiency dynamic evaluation [10,43,44]. In order to accommodate multi-period scenarios [29,43,55,58], we further extend IFDTRSs with the single period decision making to the multi-period situation. With the aid of the results of the single period decision making, we analyze the determination for the aggregation of the loss function by comparing three typical operations and then design a corresponding algorithm for deriving three-way decisions. This study provides us with a series of decision analysis methodologies when the loss functions of DTRSs are IFNs and extends the range of applications.

The remainder of this paper is organized as follows: Section 2 provides some basic concepts of Bayesian decision procedure and IFSs. Basic model of IFDTRSs is designed in Section 3. Then, decision analysis of IFDTRSs for single-period decision making and its relevant properties are explored in Section 4. With the above analysis, decision analysis of IFDTRSs for multi-period decision making is further studied in Section 5. An example is presented to illustrate the application of the novel three-way decisions in Section 6. Section 7 concludes the paper and elaborates on future studies.

2. Preliminaries

Basic concepts, notations and results of Bayesian decision procedure [6,51–53] and IFSs [1,15,40,41,45,47] are briefly reviewed in this section.