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Novel frameworks for creating robust multi-objective benchmark problems

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A B S T R A C T

Robust optimization deals with considering different types of uncertainties during the optimization process in order to obtain reliable solutions, a critical issue when solving real problems. Multiple objectives are another vital aspect of real problems that should be considered during optimization. In order to benchmark the performance of different meta-heuristics, test problems are essential, as the literature shows. Despite the significant number of studies in developing multi-objective test problems, there is currently neither study on the suitability of the current robust multi-objective benchmark problems, nor standard frameworks to create them. This motivates our attempts to investigate the features of the current robust test problems and propose three novel frameworks to generate various robust multi-objective test problems with alterable parameters. As case studies, Robust Multi-Objective Particle Swarm Optimization (RMOPSO), Robust Non-dominated Sorting Genetic Algorithm (RNSGA-II), Robust Multiobjective Evolutionary Algorithm Based on Decomposition (RMOEA/D), Robust Two Local Best Multi-objective Particle Swarm Optimization (R2LB-MOPSO), and Robust Decomposition-Based Multi-objective Evolutionary Algorithm with an Ensemble of Neighborhood Sizes (RENS-MOEA/D) are benchmarked on the proposed test problems. The results show that the proposed frameworks are able to generate robust multi-objective test problems with different adjustable characteristics and levels of difficulty. In addition, the results show that the test problems generated by the proposed frameworks can provide very challenging test beds for effectively benchmarking the performance of robust meta-heuristics.

1. Introduction

One of the key concepts in the optimization of real problems is robustness. Robust optimization refers to the process of finding optimal solutions for a particular problem that have least variability to probable uncertainties. Uncertainties are unavoidable in the real world and occur in different components of a system: operating/environmental conditions, parameters, outputs, and constraints. Such uncertainties do not usually occur in laboratories when trying to design systems.

Generally speaking, robust optimization refers to the process of considering any types of uncertainties during optimization. In the literature, this term has mostly referred to handling uncertainties in the parameters of a problem. In this work, however, we use the term "robust optimization" for considering any type of uncertainties. There are different classifications

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in the literature for categorizing uncertainties [4,35]. We utilize the classification provided by Beyer and Sendhoff [4], in which the uncertainties are categorized based on their sources, as follows:

- *Type A:* this uncertainty occurs in environmental and operating conditions. Perturbations in speed, temperature, moisture, angle of attack in airfoil design, or speed of the vehicle in propeller design are some examples of this type of uncertainty.
- *Type B:* in this case the parameters may vary after determining the optimal solution(s). One of the major sources of this kind of uncertainty is manufacturing tolerances.
- *Type C:* in this case the system itself produces noisy outputs. It might be due to sensory measurement errors or randomised simulations. Simulators that approximate outputs of systems generate this type of uncertainty. This also may happen when the evaluation of a fitness function is expensive or an analytical fitness function is not available; *e.g.* in Computational Fluid Dynamics (CFD) problems. The main difference between this type of uncertainty and type A is that the error is deterministic. Time-varying (dynamic) systems also fall under type C uncertainty.
- *Type D:* in this type of uncertainty, the constraints are perturbed (feasibility uncertainties). These uncertainties are different from the three above-mentioned uncertainties in that they affect the boundaries of the search space.

Another issue when solving real problems is that of multiple objectives. Multi-objective optimization refers to the process of considering more than one objective simultaneously when solving a problem [14]. In contrast to single-objective optimization, the ultimate goal of a multiple objective optimizer is to find a set of solutions called Pareto optimal solutions that represent the best trade-offs between the objectives. Maintaining multi-objective formulation of problems allows a designer to optimize problems with different conflicting/non-conflicting objectives across a wide range of design parameters [7]. There are many publications in this field [50–52]. In contrast to multi-objective optimization, unfortunately, robust optimization has not gained similar attention. Therefore, robust optimization (particularly for multi-objective problems) lags far behind other concepts of optimization.

Regardless of the significantly different popularity of both fields, the common tools when developing or proposing new optimization techniques are benchmark problems. Generally speaking, test problems are essential for benchmarking meta-heuristics in terms of different capabilities. In the field of multi-objective optimization, there are many studies considering benchmark problems. Various test functions have been proposed and there are sets of standard test functions. In the field of robust multi-objective optimization, however, there has been neither study of the suitability of the current test function, nor is there a set of standard test functions. This motivates our attempts to investigate the effectiveness of current robust multi-objective test problems and propose a set of new standard test functions using three novel frameworks, which provide the most challenging test beds in the literature and are able to benchmark robust multi-objective meta-heuristics from different perspectives. The rest of the paper is organized as follows.

Section 2 provides the preliminaries and essential concepts of robust optimization in both single and multi-objective search spaces. A literature review of the current robust multi-objective test problems is discussed in Section 3. Section 4 proposes three frameworks to create a standard set of robust multi-objective test functions. The experimental results of several multi-objective optimization algorithms are demonstrated in Section 5. Section 6 concludes the work and suggests some guidelines for future studies.

2. Robust optimization

As mentioned in Section 1, there are four types of uncertainty in a system, of which perturbation in the parameters can be considered as the most important one. In the following paragraphs, we discuss this type of uncertainty (Type B) in the context of multi-objective search spaces.

There is a set of robust solutions for a multi-objective problem because of its nature as discussed in Section 1. Without loss of generality, the robust multi-objective optimization considering uncertainties in the parameters is formulated as a minimization problem as follows:

$$\text{Minimize}: \quad F(\vec{x}+\vec{\delta}) = f_1(\vec{x}+\vec{\delta}),\ldots,f_o(\vec{x}+\vec{\delta}) \tag{2.1}$$

$$\text{Subject to}: \quad g_i(\vec{x}+\vec{\delta}) \geqslant 0, \quad i = 1,2,\ldots,m \tag{2.2}$$

$$h_i(\vec{x}+\vec{\delta}) = 0, \quad i = 1,2,\ldots,p \tag{2.3}$$

$$L_i \leqslant x_i \leqslant U_i, \quad i = 1,2,\ldots,n \tag{2.4}$$

where $\vec{x}$ is the set of parameters, $\vec{\delta}$ indicates the uncertainty vector corresponding to each variable in $\vec{x}$, o is the number of objective functions, m is the number of inequality constraints, p is the number of equality constraints, $[L_i, U_i]$ is the boundary of the ith variable.

In robust single-objective optimization, there is a single robust solution that might be either the global or a local optimum. The ultimate goal is to find the best solution that is not sensitive to the probable uncertainties. Since there is one comparison criterion (the objective function), the solutions can be compared easily with inequality/equality operators. In robust multi-objective optimization, however, two solutions cannot be compared with similar operators as in robust single-objective optimization. This is due to the fact that two solutions in a multi-objective search space might be incomparable