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Lagrange stability of neural networks with memristive synapses and multiple delays

Ailong Wu [a,b,c], Zhigang Zeng [c,*]

[a] College of Mathematics and Statistics, Hubei Normal University, Huangshi 435002, China
[b] Institute for Information and System Science, Xi'an Jiaotong University, Xi'an 710049, China
[c] School of Automation, Huazhong University of Science and Technology, Wuhan 430074, China

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ABSTRACT

In this paper, a general class of neural networks with memristive synapses and multiple delays is introduced and studied. Within mathematical framework of the Filippov solution, some analytical results on the Lagrange stability are established. The derived results can characterize the fundamental electrical properties of memristor devices and provide certain theoretical guidelines for applications.

1. Introduction

In the early 1970s, Professor Leon O. Chua [8] postulated the existence of a new basic electrical circuit element, called the memristor, defined by a nonlinear relationship between charge and flux linkage. It thus took its place along side the rest of the more familiar circuit elements such as the resistor, capacitor, and inductor. Chua not only foresaw the memristor but he predicted how they should behave. Although a variety of physical devices, including thermistors, discharge tubes, Josephson junctions and even ionic systems such as the Hodgkin-Huxley model of a neuron, were shown to exhibit memristive effects, a physical passive two-terminal memristive prototype could not be constructed until very recently, in 2008, members of the Hewlett–Packard Laboratories [31,32] fabricated a solid state implementation of the memristor and thereby cemented its place as the fourth circuit element. They used two titanium dioxide films, with varying resistance which is dependent on how much charge has been passed through it in a particular direction. As a result of this realization, it is possible to have nonvolatile memory on a nano scale [9,11,16,25–27,29].

Large-scale biological networks with zero, negative and positive edge weightings have become one of the most attractive research fields [3–6,30]. Recent studies also indicate that memristor bridge consisting of some identical memristors could perform zero, negative, and positive synaptic weightings. Meanwhile, the memristor bridge can mimic the functionalities of the human brain [2,18,28]. Therefore, design of simple weighting circuits for synaptic multiplication between arbitrary input signals and weights is extremely important in artificial neural systems, and it is also expected to be applicable to neural learning and sequential processing operations. In this situation, neurodynamic systems with memristive synapses have aroused the interest of numerous scholars [2,17,18,28,36–38]. Especially, the neural networks with memristive synapses

* Corresponding author. Tel.: +86 13871412009.
E-mail addresses: alequ@126.com (A. Wu), hustzgzeng@gmail.com (Z. Zeng).

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emerge the bridge-like switching weightings, and many researchers believe that such brain-like systems have high potential for neuromorphic computing. In fact, there have been some significant efforts to develop bio-inspired memristive neural networks, in which on-board training as well as parallel processing capability are implemented [2,28].

In recent years, some novel and interesting results on nonlinear systems have been reported (see [43–48], the references cited therein). It is well known that one of the important reasons why the memristor has not yet played a major role in modeling problems can most likely be explained from the fact that a plethora of complex dynamic behaviors appear even in a simple memristive system [9,16,18,29], owing to the memristor nonlinearity. And so far, it is still a difficult issue about how to develop effective methods to analyze the complex dynamics of memristive systems. Actually, the dynamic properties for memristive systems can provide the designer with an exciting variety of properties, richness of flexibility, and opportunities.

Based on above motivations, in this paper, we introduce a general class of neural networks with memristive synapses and multiple delays, and study the Lagrange stability in the presence of two different classes of feedback functions. More precisely, the contributions of this paper are described below:

(1) The neural network model with memristive synapses discussed in this paper is a state-dependent switched nonlinear system, which is the generalization of those for conventional neural networks. Therefore, the obtained results can be used in the wider scope.
(2) We consider the Lagrange stability. It is noted that unlike Lyapunov stability, Lagrange stability refers to the stability of the total system, not the stability of the equilibriums, because the Lagrange stability is considered on the basis of the boundedness of solutions and the existence of global attractive sets. Moreover, with regard to Lagrange stability, outside the global attractive sets, there is no equilibrium point, periodic state, almost periodic state, or chaos attractor [19,20,24,33–35,39]. Undoubtedly, these exclusive dynamical properties of Lagrange stability are absolutely crucial to memristive systems, due to their instantaneous complex memristive synaptic operations.
(3) The method proposed in this paper can be applied to analyze the multi-attractors of other classes of switched network clusters. These issues will be the topics of future research.

The remaining part of the paper consists of four sections. In Section 2, some preliminaries are introduced. In Section 3, the main results are derived. Several illustrative examples are given to demonstrate the effectiveness and performance of the obtained results in Section 4. Finally, concluding remarks are included in Section 5.

2. Preliminaries

2.1. Model

In this paper, referring to some relevant works in [17,18,36–38], which deal with the detailed construction of some general classes of memristive neurodynamic systems from the aspects of circuit analysis and memristor physical properties, we consider a class of neural networks with memristive synapses and multiple delays of the form

$$\dot{x}_i(t) = -x_i(t) + \sum_{j=1}^{n} a_{ij}(x_i(t)) f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij}(x_i(t)) f_j(x_j(t-\tau_j)) + u_i, \quad t \geqslant 0, \quad i = 1,2,\ldots,n, \tag{1}$$

where $x_i(t)$ denotes the memristor state variable, τ_i is the transmission delay and satisfies $0 \leqslant \tau_i \leqslant \tau$ (τ is a constant), u_i denotes external input, $f_i(\cdot)$ is feedback function, $a_{ij}(x_i(t))$ and $b_{ij}(x_i(t))$ represent memristive synapse weights, which are defined as

$$a_{ij}(x_i(t)) = \begin{cases} \hat{a}_{ij}, & |x_i(t)| < \Upsilon_i, \\ \check{a}_{ij}, & |x_i(t)| > \Upsilon_i, \end{cases} \quad b_{ij}(x_i(t)) = \begin{cases} \hat{b}_{ij}, & |x_i(t)| < \Upsilon_i, \\ \check{b}_{ij}, & |x_i(t)| > \Upsilon_i, \end{cases} \tag{2}$$

for $i, j = 1,2,\ldots,n$, where the switching jumps $\Upsilon_i > 0$, $\hat{a}_{ij}$, $\check{a}_{ij}$, $\hat{b}_{ij}$, and $\check{b}_{ij}$ are constants.

Remark 1. The features of pinched hysteresis memristor are complex and diversified, and Fig. 1 shows the switching current–voltage characteristics. Hu and Wang [17] discuss the convergent behavior of memristive neurodynamic systems under the forward and reverse current–voltage operating mechanisms. Wen and Zeng [36] investigate a class of memristive neurodynamic systems under the bounded current–voltage switching mechanism. In fact, even $a_{ij}(x_i(t))$ and $b_{ij}(x_i(t))$ appear other types of memristive twinkling changes, the main results in this paper still can be made some parallel promotions. Here we consider a preliminary neurodynamic system (1) in the presence of multipolar memristive pinched hysteresis (2), which may be a good candidate for the realization of innovative oscillatory associative and dynamic memories based on simple topological structure.

The initial conditions of network (1) are assumed to be

$$x_i(s) = \psi_i(s), \quad s \in [-\tau, 0], \quad i = 1,2,\ldots,n,$$

where $\psi_i(s)$ is a continuous function defined on $[-\tau, 0]$.