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# Data-based robust optimal control of continuous-time affine nonlinear systems with matched uncertainties☆

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ABSTRACT

In this paper, the robust optimal control of continuous-time affine nonlinear systems with matched uncertainties is investigated by using a data-based integral policy iteration approach. It is a natural extension of the traditional optimal control design, under the framework of adaptive dynamic programming (ADP) method, to robust optimal control of nonlinear systems with matched uncertainties. In theoretical aspect, by increasing a feedback gain to the optimal controller of the nominal system, the robust controller of the matched uncertain system is obtained, which also achieves optimality with a newly well-defined cost function. When regarding the implementation, the data-based integral policy iteration algorithm is used to solve the Hamilton–Jacobi–Bellman equation corresponding to the nominal system with completely unknown dynamics information. Then, the actor-critic technique based on neural networks and least squares implementation method are employed to facilitate deriving the optimal control law iteratively, so that the closed-form expression of the robust optimal controller is available. Additionally, two simulation examples with application backgrounds are presented to illustrate the effectiveness of the established robust optimal control scheme. In summary, it is important to note that the result developed in this paper broadens the application scope of ADP-based optimal control approach to more general nonlinear systems possessing dynamical uncertainties.



## 1. Introduction

The phenomenon of dynamical uncertainties is common in practical control systems. From the literature of modern nonlinear control, it is known that the presence of dynamical uncertainties makes the feedback control problem extremely challenging in the context of nonlinear systems. As a result, the problem of designing adaptive and robust controller for nonlinear systems with uncertainties has attained considerable attention [5,10,13,16,24,33–35,38,45,51–53,57]. Among them, Mu et al. [35] proposed a general design scheme of finite-time switching mode manifolds and corresponding nonsingular

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controllers. Yang et al. [57] proposed a networked-predictive-control scheme to compensate for the network-induced delay, so that the problem of output feedback controller design for networked control systems with mixed communication delays can be addressed. These results are beneficial to the development of the modern control theory. Additionally, Lin et al. [24] showed that the robust control problem can be solved by means of studying the optimal control problem of the related nominal system, but the detailed procedure was not presented in that paper. Wang et al. [45] constructed a novel strategy to achieve robust stabilization for a class of uncertain nonlinear systems based on the online policy iteration algorithm. However, the optimality of robust controller with respective to a newly defined cost function was not taken into consideration. Moreover, the complete system dynamics, which were usually difficult to obtain for practical systems, were required during the algorithm implementation. To the best of our knowledge, there are no results on designing robust optimal control for uncertain nonlinear systems by using a data-based policy iteration approach. This is the motivation of our research. Actually, it is the first time to establish the robust optimal control method for a class of nonlinear systems possessing uncertainties via a data-based integral policy iteration learning technique with completely unknown dynamics.

Nowadays, the data-based control design has become a hot topic in the field of control theory and control engineering [12,29]. In this paper, the starting point of the obtained strategy is the data-based optimal control design. Note that studying the nonlinear optimal control problem always requires to solve the Hamilton–Jacobi–Bellman (HJB) equation. Though dynamic programming has been a classical method in solving optimization and optimal control problems, it often encounters the phenomenon of “curse of dimensionality” [2]. For avoiding this difficulty, adaptive/approximate dynamic programming (ADP) was introduced by Werbos [50] and Prokhorov and Wunsch [39] as an effective method to solve the optimal control problem forward-in-time, based on function approximation structures [7,8,21,25,32,42,46], such as neural networks, support vector machine, fuzzy logic, etc. Reinforcement learning is another computational method which can interactively find an optimal policy from the learning process between the agent and the environment. Remarkably, Lewis and Liu [20], and Lewis and Vrabie [21] have given some opinions that the idea of ADP is very closely related to the framework of reinforcement learning. Recently, the researches on ADP and reinforcement learning have gained much attention from scholars of numerous fields [3,6,11,23,25–28,30,32,36,37,41,42,44,46–49,54–56,58–62]. Among the various results, robust ADP was developed for the design of robust optimal controllers for linear and nonlinear systems subject to both parametric and dynamic uncertainties by Jiang and Jiang [16], in order to broaden the application scope of ADP theory in the presence of dynamic uncertainties. In addition, Jiang and Jiang [13] also extended the robust ADP approach to decentralized optimal control of a class of large-scale systems with uncertainties. Note that in [16], the control signal of the nonlinear system was only one-dimensional and the optimization issue with regard to the original uncertain nonlinear system was not presented, while in [13], the robust decentralized control approach was only suitable for a class of linear systems. These inevitably restrict the effect of the proposed methods to some extent.

In the existing literature of ADP-based optimal control, either policy iteration or value iteration is employed to solve the Bellman equation or the HJB equation. The information of control matrix is necessary when employing the traditional policy iteration algorithms. However, in many situations, it is difficult to acquire the accurate model of controlled plant. The ADP and reinforcement learning schemes, which have the learning and optimization capabilities, can relax the requirement for a complete and accurate model of the controlled plant, by virtue of considering compact parameterized function representations whose parameters can be adjusted through learning and adaption. Jiang and Jiang [15] presented a novel policy iteration approach for continuous-time linear systems with completely unknown dynamics. Vrabie and Lewis [43] derived an integral reinforcement learning method to obtain direct adaptive optimal control for nonlinear input-affine continuous-time systems with partially unknown dynamics. Lee et al. [18,19] presented an integral reinforcement learning algorithm for continuous-time systems without the exact knowledge of the system dynamics. Liu et al. [25] developed a neural-network-based decentralized control strategy of a class of continuous-time nonlinear interconnected systems without requirement of dynamical information. Bian et al. [4] proposed a novel optimal control design approach for continuous-time nonaffine nonlinear systems with unknown dynamics by the idea of ADP. However, the system uncertainties were not considered in the above results.

With this background, how to further extend the application scope of ADP approach to more general nonlinear systems with dynamic uncertainties arouses our wide concern. In this paper, we investigate the data-based robust optimal control of continuous-time nonlinear systems with matched uncertainties. To begin with, the problem statement and some preliminaries are provided. It is proved that the improvement of the optimal control law is nothing but the robust controller of the original uncertain system, which also attains the property of optimality with a newly defined cost function. This serves as the main theoretical result of the paper. Then, the optimal controller of the nominal system is obtained by the data-based integral policy iteration algorithm and the neural network technique with completely unknown system dynamics, which is regarded as the primary implementation procedure. At last, two simulation examples are given to show the good response performance of the present robust optimal control scheme.

## 2. Problem statement and preliminaries

In this paper, we study a class of continuous-time nonlinear systems with input-affine structure and matched uncertainties described as

$$\dot{x}(t) = f(x(t)) + g(x(t))\Big(\bar{u}(t) + \bar{d}(x(t))\Big), \tag{1}$$