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Preference structures: Qualitative judgements based on smooth t-conorms

I. Aguiló [a], T. Calvo [b,*], P. Fuster-Parra [a], J. Martin [a], G. Mayor [a], J. Suñer [a]

[a] Departament de Ciències Matemàtiques i Informàtica, Universitat de les Illes Balears, 07122 Palma de Mallorca, Spain
[b] Departamento de Informática, Universidad de Alcalá, 28871 Alcalá de Henares, Madrid, Spain

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ABSTRACT

The problem of the consistency of qualitative judgements about the difference of attractiveness between alternatives is studied. Our proposal deals with smooth t-conorms to be used for aggregation of elementary judgements.

1. Introduction

A preference structure on a set of alternatives A is a triplet (P, I, J) of binary relations in A defining a decision maker's preferences, as follows: aPb if and only if the decision maker prefers a to b; aIb if and only if the decision maker is indifferent between alternatives a and b, and aJb if and only the decision maker is unable to compare a and b. These structures are well-studied mathematical structures in the theory of preference modeling.

In decision making under uncertainty, the construction of appropriate models to represent the preferences of decision makers is needed. Decision models are traditionally quantitative but, recently, qualitative models have been developed in order to construct models closer to the natural language of decision maker opinions (see [2,6] and [7]). In this way, our approach is based on providing preferential information about two alternatives at a time, firstly by giving a judgement as to their relative attractiveness (ordinal judgement) and secondly, if the two alternatives are not deemed to be equally attractive, by expressing a qualitative judgement about the difference of attractiveness between the most attractive of the two alternatives and the other (see [13]).

Among the existing methodologies concerning multicriteria decision making, it could be mentioned AHP (Analytic Hierarchy Process) developed by Saaty in the 1970s (see [19]), and MACBETH (Measuring Attractiveness by a Categorical Based Evaluation Technique) (see [3]). The AHP is a structured technique for organizing and analyzing complex decisions, based on mathematics and psychology. In AHP the user decomposes the decision problem into a hierarchy of more easily comprehended sub-problems, then the decision maker evaluates these different parts by comparing each of them with each other

* Corresponding author. Tel.: +34 91 8856653.
E-mail addresses: isabel.aguilo@uib.es (I. Aguiló), tomasa.calvo@uah.es, tomasa.calvo@gmail.com (T. Calvo), pilar.fuster@uib.es (P. Fuster-Parra), javier.martin@uib.es (J. Martin), gmayor@uib.es (G. Mayor), jaume.sunyer@uib.es (J. Suñer).

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two by two; finally the AHP converts these evaluations to numerical weights or priorities. The AHP methodology has been widely applied (see [21,22]). Most of the criticisms related to AHP involve a non–desirable phenomenon called *rank reversal*, that is, when new alternatives are added to a decision problem, the ranking of the old alternatives changes.

MACBETH requires only qualitative judgements about differences of values, to help an individual or a group quantify the relative attractiveness of options. As the qualitative judgements of an evaluator are entered in MACBETH, the software automatically verifies their consistency, and suggests potential solutions to solve possible inconsistencies. In [3] six semantic categories of difference of attractiveness are used: *very weak*, *weak*, *moderate*, *strong*, *very strong*, and *extreme*.

Fundamental references on this topic are [8] and [17] where applications of Measurement Theory to problems of Decision Making can be found.

The main objective of this paper is to propose a methodology for obtaining differences of attractiveness between alternatives. This methodology consists of building differences of attractiveness from the ones for consecutive classes of indifferent alternatives by using smooth t-conorms as aggregator. In this way, only these differences of attractiveness are required, since then the whole set of differences of attractiveness is automatically obtained from them. Moreover, we prove that the use of smooth t-conorms ensures the consistency of the resulting preference inequalities.

This contribution is organized as follows. Section 2 is devoted to introduce basic definitions and results used along the paper. In Section 3 we deal with basics of preference structures and the problem of their cardinal representation. Section 4 shows how a valued preferential information can be obtained by means of a questioning procedure. Section 5 contains the main results of the paper. They deal with the use of smooth t-conorms to perform the aggregation of elementary judgements. Finally, in Section 6 we analyze some relationships between our approach and AHP and MACBETH methods. An Appendix includes an Algorithm to compute the consistency of any preference linear inequalities system.

2. Preliminaries

A binary relation R on a given set A is a subset of the Cartesian product $A \times A$. If (a, b) belongs to R then both the notations $(a, b) \in R$ or aRb are used indifferently. Note that a binary relation R on A can be also defined as a function $R: A \times A \to \{0, 1\}$. Thus we can define L-valued binary relations R on A by considering functions $R: A \times A \to L$, where L is a more general set of values. In the case $L = [0, 1]$ we say that R is a fuzzy binary relation on A. When $L = \{0, 1\}$ we also say that R is a crisp binary relation on A.

We will use the expression xR^cy instead of $(x, y) \notin R$, and R^{-1} is the binary relation on A defined by $xR^{-1}y$ if and only if yRx (the inverse of R).

First we recall some basic properties that could be required for a binary relation R on A. For all $x, y, z \in A$:

- Reflexive: xRx,
- Irreflexive: xR^cx,
- Symmetric: $xRy \Rightarrow yRx$,
- Asymmetric: $xRy \Rightarrow yR^cx$,
- Antisymmetric: xRy and $yRx \Rightarrow x = y$,
- Transitive: xRy and $yRz \Rightarrow xRz$,
- Negatively transitive: xR^cy and $yR^cz \Rightarrow xR^cy$,
- Complete (Linear): xRy or yRx.

A binary relation R on A is called:

- Equivalence relation: if R is reflexive, symmetric and transitive,
- Preorder: if R is reflexive and transitive,
- (Partial) Order: if R is reflexive, antisymmetric and transitive.

Despite the fact that t-norms and t-conorms were first introduced in the context of statistical metric spaces, they have become an important tool in many other fields: fuzzy sets, decision making, statistics, theories of non-additive measures, etc. Comprehensive monographs on t-norms and t-conorms are [1] and [14]. According to the fact that in most practical situations it is necessary to discretize the real unit interval, we need to deal with logics where the set of truth values is modeled by a finite linearly ordered set $L = \{0, 1, \ldots, n\}$. As we mentioned before, we will deal with t-conorms. As it is expected, in the definition of discrete triangular conorms (discrete t-conorms, for short) we use the set of axioms provided by Schweizer and Sklar [20] once adapted to this finite setting. Thus, our requirements on a t-conorm $S: L \times L \to L$ for all a, b, c, d in L are:

(i) $S(a, b) = S(b, a)$,
(ii) $S(S(a, b), c) = S(a, S(b, c))$,
(iii) $S(a, b) \leq S(c, d)$ whenever $a \leq c$ and $b \leq d$,
(iv) $S(a, 0) = a$.

The following are the three basic discrete t-conorms:

- $S_M(a, b) = \max(a, b)$, (maximum)