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Topology selection for particle swarm optimization☆

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ABSTRACT

Particle swarm optimization (PSO) uses a social topology for particles to share information among neighbors during optimization. A large number of existing literatures have shown that the topology affects the performance of PSO and an optimal topology is problem-dependent, but currently there is a lack of study on this issue. In this paper, we first analyze a class of deterministic regular topologies with regard to what affect the optimality of algorithmic parameters (e.g., the number of particles and the topological degree), so as to provide a guide to topology selections for PSO. Both theoretical analysis and numerical experiments are performed and reported in detail. The theoretical analysis reveals that the optimality of algorithmic parameters is dependent on the computational budget available. In particular, the optimal number of particles increases unstrictly as the computational budget increases, while for any fixed number of particles the optimal degree decreases unstrictly as computational budget increases. The only condition is that the computational budget cannot exceed a constant measuring the hardness of the benchmark function set. With a total of 198 regular topologies and 9 different numbers of particles tested on 90 benchmark functions using a recently reported data profiling technique, numerical experiments verify the theoretical derivations. Based on these results, two formulas are developed to help choose optimal topology parameters for increased ease and applicability of PSO to real-world problems.

1. Introduction

During the recent decades, a significant and rapidly increasing amount of real-world problems have been solved using swarm intelligence algorithms such as genetic algorithm [16], particle swarm optimization [26], ant colony optimization [9], firefly algorithm [13,56], fireworks algorithm [49], and brain storm optimization [46]. This paper considers a particular type of swarm intelligence algorithm, the particle swarm optimization (PSO) algorithm, which was developed by mimicking bird flocking behaviors [11,26]. Despite PSO's successes across a wide range of applications [1,14,41,53,54,60], many variants have been proposed to improve performance [4,7,12,17,25,27,28,30,31,37,45,57,58]. Further, theoretical analyses have been undertaken to optimize the configuration of the algorithm, especially its topology selection [28,36], collective dynamic behavior [51] and parameter setting [2,5,7,20,22,35,42,50,52].

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In this paper, we focus on the issue of topology selection for PSO, because this aspect proves to be the most challenging in optimizing the algorithm [28,36,43]. The topology determines each particle's neighbors. During the evolutionary process of PSO, each particle shares its best experiment with its neighbors and learns from its neighbors' best positions. Therefore, the topology is important for the overall PSO performance [28,36].

The global best (gbest) topology and the local best (lbest) topology are two common social topologies in PSO. In the gbest topology, each particle is connected with all other particles, i.e., it is a fully connected graph. In the lbest topology, each particle is only connected with its nearest K neighbors. In most publications and in this paper, $K = 2$ is used without loss of generality.

In order to optimize topology, several works have been dedicated to understanding how different topologies affect PSO performance. In [23], for example, several social topologies, including cycles, wheels, stars and some random graphs with or without small-world modifications, have been tested on a standard suite of test functions. Further explorations have also been reported in [28] and [36], where 1343 topologies and 3289 topologies were tested, respectively. These studies confirm that PSO performance is significantly affected by its topology, and the von Neumann topology is relatively superior in the sense of a higher proportion of successes and lower standardized performance. However, not a single candidate topology has outperformed all others on all test functions [28]. In short, an optimal topology is so far problem-specific, and different topologies are often required for different problems.

A conclusion drawn from [28] and [36] is that it is hard to balance local search and global search with one topology. A random topology [28,36] is hard to offer optimal topological parameters (i.e., graph statistics) and statistical regularities have not emerged as a small number of test functions have been used so far. Moreover, there exist certain important issues that remain to be addressed. For example, both [28] and [36] have indicated that a small degree (about 3–5) of the topology performs well, such as the von Neumann topology with a degree of 4, but no reasons have been given or sustained so far.

Based on the above observations, we attempt to answer the following two questions in this paper:

- Given the number of particles m, what is the optimal degree of the PSO topology and how this may be determined?
- Further, what is the optimal number of particles and how this may be determined ?

To seek answers to these questions, we focus on a representative benchmark set and test various topology strategies. First, we shall adopt a class of deterministic topologies, which includes gbest and lbest topologies as special cases. Second, we shall adopt a set of benchmark functions that is large enough to yield statistical significance. Third, we shall measure the performance with the "proportion of functions solved", extending from the "proportion of successes", so as to measure both "short-run performance" and "long-run performance" more rigorously.

In order to verify and supplement the theoretical analysis, a total of 198 regular topologies with 9 different numbers of particles will be tested on 90 benchmark functions. The data profiling technique proposed recently in [40] is adopted in analyzing the large amount of data generated from the experiments. The results will help formulate topology selection for PSO.

The rest of this paper is organized as follows. In Section 2, the PSO algorithm is outlined and briefly reviewed. In Section 3, a regular form of PSO topology is defined, its graph statistics are computed, an algorithm to generate the regular topology is proposed, pre-optimal theoretical analyses are derived with a computational budget. In Section 4, extensive numerical experiments are undertaken. Conclusions are drawn in Section 5.

2. PSO and its static topology optimization

In this section, we first outline and then briefly review the canonical PSO and existing work on its static topology optimization, with results and methodologies discussed. PSO with an inertia weight [45], which is equivalent to PSO with the constriction coefficient [7], is regarded as canonical PSO in this paper.

2.1. Particle swarm optimization

In a PSO algorithm, consider a swarm containing m particles, each of them possesses an initial position $\boldsymbol{x}_i(0)$ and velocity $\boldsymbol{v}_i(0)$, where $i = 1, \ldots, m$ and the parenthesis index represents an iteration of the swarm. Then each particle i communicates with other particles in its neighborhood N_i, and changes its velocity and position in dimension j at iteration k according to the following dynamic equation

$$\boldsymbol{v}_{ij}(k+1) = \omega \boldsymbol{v}_{ij}(k) + C_1(\boldsymbol{p}_{ij}(k) - \boldsymbol{x}_{ij}(k)) + C_2(\boldsymbol{g}_{ij}(k) - \boldsymbol{x}_{ij}(k)), \tag{1a}$$

$$\boldsymbol{x}_{ij}(k+1) = \boldsymbol{x}_{ij}(k) + \boldsymbol{v}_{ij}(k+1), \tag{1b}$$

where $i = 1, \ldots, m$, $j = 1, \ldots, n$, $k = 0, 1, \ldots,$ and n is the dimension of the objective function $f(x)$. $\boldsymbol{p}_i(k)$ and $\boldsymbol{g}_i(k)$ are defined by

$$\boldsymbol{p}_i(k) = \arg\min_{0 \le t \le k} f(\boldsymbol{x}_i(t)), \quad \boldsymbol{g}_i(k) = \arg\min_{l \in N_i} f(\boldsymbol{p}_l(k)), \tag{2}$$

and are often called the personal best (position) and the neighborhood best (position) of particle i, respectively.