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Optimal supervisor synthesis for petri nets with uncontrollable transitions: A bottom-up algorithm[☆]

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ABSTRACT

Petri nets are a widely used tool to model, analyze and control discrete event systems that arise from automated production, intelligent transportation, and workflow management. For a class of Petri nets with uncontrollable transitions, this paper proposes a bottom-up algorithm to transform a given generalized mutual exclusion constraint into an optimal admissible one. Based on the transformation, a design method is proposed to synthesize an optimal supervisor. Compared with the existing methods that require the computation of exponential complexity, the proposed one can obtain an optimal supervisor with polynomial complexity.

1. Introduction

A generalized mutual exclusion constraint (GMEC) is a condition that limits a weighted sum of tokens contained in a set of places. A GMEC can describe many DES control problems [1–61] and thus fulfilling the forbidden state specifications represented by GMEC is a hot topic [3,8,9,12,16,20,26–36,59–61]. This work deals with the forbidden state problems that can be represented by GMEC.

In a Petri net with uncontrollable transitions, it is possible that a forbidden marking may be reached from a legal marking by firing only uncontrollable transitions. Many researchers have studied the problem of restricting a Petri net's evolution within the admissible marking set [1,10,11,13,25,38,42].

By utilizing the graphical representation of the state transition logic, Holloway and Krogh [6] synthesize an optimal supervisor to enforce a class of GMEC on cyclic marked graphs. Kumar and Holloway [42] develop an algorithm for computing a minimally restrictive control policy when the plant behavior is a deterministic Petri net language and the desired behavior is a regular language. Li and Wonham [14,15] point out that the computation of the admissible marking set can be reduced to a linear programming problem (LPP) if the uncontrollable subnet is loop-free. Based on LPP, they propose a method of synthesizing an optimal supervisor. For backward conflict-free choice nets (BCFCN), Basile et al. [3] synthesize an optimal supervisor based on minimal T-invariants and LPP. The significance of their work lies in the fact that an optimal supervisor can be obtained. However, these approaches suffer from the problem of computational complexity since a significant online computational effort is required.

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Moody and Antsaklis [22] propose the concept of admissible GMEC and transform a given GMEC into an admissible one by performing row operations on a matrix containing the uncontrollable columns of the plant incidence matrix. Based on constraint transformation, they propose a supervisor synthesis technique for Petri nets with uncontrollable transitions. Their method is very efficient from a computational point of view but it can obtain one admissible GMEC only and thus fails to find the optimal one. Basile et al. [2] extend it by adding two parameters to the matrix containing the uncontrollable columns of the plant incidence matrix. Applying their method can lead to more than one admissible GMEC and thus a suboptimal one can be obtained.

Based on S-decrease, Chen [5] proposes a control synthesis method that is proved to be optimal when the uncontrolled subnet of a net is a generalized state machine or generalized marked graph. It shows in [5] that the S-decrease-based control synthesis method is in many cases better than that in [22].

For a class of Petri nets in which the uncontrollable influence subnets are forward synchronization and forward conflict free (FSFCF) nets, Luo et al. [18] propose a method to transform a given GMEC into an optimal admissible one based on a crux path set. For another kind of uncontrollable influence subnets called forward synchronization and backward conflict free (FSBCF) nets, Luo et al. [19] propose a necessary and sufficient condition for the existence of an optimal supervisor and a method to synthesize it based on crux path sets and constraint transformation. Based on the results of [18] and [19], Luo et al. [20] extend the crux path set-based constraint transformation approach to a more general class of uncontrollable influence subnets called forward-concurrent-free nets. The constraint transformation approach based on crux path sets can obtain an optimal admissible GMEC given a GMEC but it is not efficient from a computational point of view since in the worst case the number of paths grows exponentially with the size of an influence uncontrollable subnet.

In order to improve the computational efficiency of the methods in [18–20], this work proposes a new approach for a class of Petri net whose uncontrollable influence subnets are FSFCF nets. Its new contributions are:

(1) The net structure of an FSFCF net and its related properties are studied and new properties are revealed, which play a critical role in the constraint transformation.
(2) Based on the newly discovered properties, a bottom-up algorithm with polynomial complexity is proposed to transform a given GMEC into an optimal admissible one; and
(3) A method is proposed to synthesize the optimal supervisor based on the transformation.

Compared with the existing methods [2,3,5,6,14,15,18–20,22], the proposed one can obtain an optimal supervisor with high computational efficiency when it is used in a class of Petri nets whose uncontrollable influence subnets are FSFCF nets.

2. Preliminaries

2.1. Ordinary petri nets [23,37]

An *ordinary Petri net* is a 3-tuple $N = (P, T, F)$ where P and T are finite, nonempty, and disjoint sets. P is a set of *places*, and T is a set of *transitions*. $F \subseteq (P \times T) \cup (T \times P)$ is the *flow relation*. Given a net $N = (P, T, F)$, and a node $x \in P \cup T$, ${}^{\bullet}x = \{y \in P \cup T \mid (y, x) \in F\}$ is its *preset*, while $x^{\bullet} = \{y \in P \cup T \mid (x, y) \in F\}$ is its *post-set*. $\forall X \subseteq P \cup T$, ${}^{\bullet}X = \cup_{x \in X}{}^{\bullet}x$, and $X^{\bullet} = \cup_{x \in X}x^{\bullet}$. $\forall x \in P \cup T$. ${}^{\bullet\bullet}x = {}^{\bullet}({}^{\bullet}x)$, and $x^{\bullet\bullet} = (x^{\bullet})^{\bullet}$. The *incidence matrix* of N is a matrix $[N]$: $P \times T \to \{-1, 0, 1\}$ indexed by P and T such that $[N](p, t) = -1$ if $p \in {}^{\bullet}t \backslash t^{\bullet}$; $[N](p, t) = 1$ if $p \in t^{\bullet} \backslash {}^{\bullet}t$; otherwise $[N](p, t) = 0$ for all $p \in P$ and $t \in T$. For a place p (transition t), its *incidence vector*, i.e., a row (column) in $[N]$, is denoted by $[N](p, \bullet)$ ($[N](\bullet, t)$).

A *marking* m of N is a mapping from P to $\mathbb{N}$ where $\mathbb{N} = \{0, 1, 2, \ldots\}$. To save space, we use multi-set notation $\sum_{p \in P} m(p)p$ to denote m, where $m(p)$ indicates the number of tokens in p at m. For example, $m = [1, 2, 0, 0]^T$ is denoted by $m = p_1 + 2p_2$. p is *marked* by m if $m(p) > 0$.

A transition t is *enabled* at m, denoted by $m[t>$, if $\forall p \in {}^{\bullet}t$, $m(p) > 0$. An enabled transition t at m can fire, resulting in a new marking m', denoted by $m\,[t>m'$, where $m'(p) = m(p) + [N](p, t)$, $\forall p \in P$. A sequence of transitions $\alpha = t_1 t_2 \ldots t_k$, $t_i \in T$, ($i \in \{1, 2, \ldots, k\}$) is feasible from a marking m_1 if there exist m_{i+1}, $i \in \{1, 2, \ldots, k\}$, such that $m_i[t_i>m_{i+1}$, $i \in \{1, 2, \ldots, k\}$. In such a case, we use $m_1[\alpha>m_{k+1}$ (or $m_1 \xrightarrow{\alpha} m_{k+1}$) to denote the case that m_{k+1} is reachable from m_1 after firing a sequence of transitions α. Let $R(N, m_0)$ denote the set of all reachable markings of N from the initial marking m_0.

A string $x_1 x_2 \ldots x_n$ is called a *path* of N if $\forall i \in \{1, 2, \ldots, n-1\}$, $x_{i+1} \in x_i^{\bullet}$, where $x_i \in P \cup T$. A *circuit* is a path in which the first and last nodes are identical. A node may appear more than once in a circuit. An *elementary path* from x_1 to x_n is a path whose nodes are all different (except, perhaps, x_1 and x_n). A path $x_1 x_2 \ldots x_n$ is called an *elementary circuit* if it is an elementary path with $x_1 = x_n$. If there exists a path from x_i to x_j, we say that x_j is reachable from x_i. By default, every node is reachable from itself. An *uncontrollable path* is a path in which each transition is uncontrollable.

A transition without any input place is called a *source transition*, and one without any output place is called a *sink transition*. Note that the former is unconditionally enabled, and that firing the latter consumes tokens, but does not produce any. A place without any input transition is called a *source place*, and one without any output transition is called a *sink place*.

2.2. Controlled petri nets [7,20]

A *controlled Petri net* (CtlPN) is a triple $N_c = (N, P_c, F_c)$ where $N = (P, T, F)$ is an ordinary Petri net structure, P_c is a finite set of *control places* with $P_c \cap P = \emptyset$, and $F_c \subseteq (P_c \times T)$ is a set of directed arcs connecting control places to transitions. In the CtlPN