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Consistency analysis and group decision making based on triangular fuzzy additive reciprocal preference relations

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ABSTRACT

Triangular fuzzy numbers are effective in modeling imprecise and uncertain information, and have been widely applied in decision making. This paper uses a cross-ratio-expressed triplet to characterize a positive triangular fuzzy number, and introduces notions of cross-ratio-expressed triangular fuzzy numbers (CRETFNs) and triangular fuzzy additive reciprocal preference relations (TFARPRs). We present transformation methods between TFARPRs and triangular fuzzy multiplicative reciprocal preference relations, and develop operational laws of CRETFNs, such as complement, addition, multiplication and power. A cross-ratio-expressed triangular fuzzy multiplication based transitivity equation is established to define multiplicative consistency of TFARPRs. The new consistency captures Tanino's multiplicative consistency among the cross-ratio-expressed modal values, and geometric consistency of the interval fuzzy preference relation constructed from lower and upper support values of cross-ratio-expressed triangular fuzzy judgments. Some desirable properties are furnished for multiplicatively consistent TFARPRs. We propose a cross-ratio-expressed triangular fuzzy weighted geometric operator to aggregate CRETFNs, and extend it to fuse TFARPRs. Score and uncertainty index functions are defined and employed to devise a novel comparison method for CRETFNs. A detailed procedure is put forward to solve group decision making problems with TFARPRs. Six numerical examples are provided to illustrate the validity and applicability of the proposed models.

1. Introduction

Group decision making comprises multiple decision makers with a common goal on deriving a rank of alternatives or selecting the best one(s) from multiple alternatives. To reach this goal, each decision maker has to express his/her preference information via a set of estimations over alternatives [8,38]. The pairwise comparison is a practical and effective method used in the preference elicitation [39,52]. In the classical analytic hierarchy process (AHP) introduced by Saaty [41], pairwise judgments are expressed as ratio-based numerical values and structured by multiplicative preference relations. However, these numerical values may not be adequate to model decision makers' preferences with vagueness and indeterminacy [17,19,32,37,64]. To address this obstruction, the fuzzy set [67] and its extensions [15] have been adopted in preference modeling. Consequently, different kinds of preference relations and fuzzy AHP methods have been developed to solve real-world decision making problems [3,4,6,16,30,35,36,44,62,63].

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Various fuzzy numbers, such as triangular fuzzy numbers (TFNs), trapezoidal fuzzy numbers, interval fuzzy numbers and type-2 fuzzy numbers, have been proposed and used to develop different decision methods and aid tools [17,34,37,45,46]. Among these fuzzy numbers perhaps TFNs are one of the most widely used in preference modeling and decision making because they are suitable to associate with linguistic terms [25,44,48,50,56], and their membership functions can be designed to meet different uncertainty levels of judgments and manage linguistic information with multi-granularity in qualitative group decision making [65]. Van Laarhoven and Pedrycz [44] defined operations on TFNs and introduced triangular fuzzy multiplicative reciprocal preference relations (TFMRPRs), in which pairwise judgments with vagueness are characterized by TFNs and the indifference between two alternatives is expressed as a TFN denoted by the triplet $(1, 1, 1)$. The research has been carried out on consistency of TFMRPRs [3,33,47,50] and priority methods [4,44,49,50] as well as group decision making with TFMRPRs [9,16,32,48,56]. For instance, Buckley [3] defined multiplicatively consistent TFMRPRs by introducing an approximate transitivity equation. Based on three multiplicative preference relations constructed from a TFMRPR, Liu et al. [33] proposed another consistency definition for TFMRPRs. Wang [50] put forward a triangular fuzzy multiplication based consistency definition for TFMRPRs, and developed a logarithmic least square model to derive triangular fuzzy multiplicative weights from TFMRPRs. Wang and Chin [49] developed a linear goal programming model to obtain priority weights of TFMRPRs. Chen et al. [9] employed TFMRPRs and fuzzy AHP to evaluate teaching performance.

A TFMRPR may be reduced to a multiplicative preference relation [41]. From this viewpoint, the elements in a TFMRPR are provided by a decision maker based on a bipolar scale with the neutral value 1, such as Saaty's 1–9 scale, and the [1/7, 7] scale.

Another usual bipolar scale is the unit interval having the neutral element 0.5. This scale is widely used in decision modeling based on [0, 1]-valued additive reciprocal preference relations [11,13] (also called fuzzy preference relations [35]), which have been widely explored in the literature [2,6,11,13,14,21,22,31,42,60,61] and extended to interval additive reciprocal preference relations [20,23,54,55,59,62] and intuitionistic fuzzy preference relations [26-30,53,57,63]. In this approach, [0, 1]-valued scales are adopted to represent intensities of preferences. The drawback of [0, 1]-valued additive reciprocal preference relations is that their elements are precise values [18], implying that it is difficult to employ them to model imprecise judgments. On the other hand, there exists an isomorphism between the bipolar unit interval with the neutral element 0.5 and the bipolar positive real line with the neutral element 1. This isomorphism indicates that the values 0 and 1 in the unit interval play the same roles as 0 and $+\infty$ in the positive real line, respectively. These lead to our research on triangular fuzzy additive reciprocal preference relations (TFARPRs) whose elements are cross-ratio-expressed triangular fuzzy numbers (CRETFNs).

Operational laws play an essential and important role for exploring consistency and aggregation of preference relations. Triangular fuzzy arithmetic operations introduced by Van Laarhoven and Pedrycz [44] are defined for TFNs denoted by real-number-expressed triplets, and are appropriate to TFMRPRs. However, the elements in a TFARPR are characterized by CRETFNs. This leads to our research on operational laws of CRETFNs. This paper puts forward operations on CRETFNs, such as complement, addition, multiplication and power, and presents their properties.

Consistency of preference relations is usually modeled and captured by transitivity among pairwise judgments. A number of researchers have been paying their attention to the study of consistency for diverse types of preference relations [3,11,13,14,21,23,24,26,28,31,33,41,42,47,50–53,55,57,61,62]. Wang and Chen [47] employed fuzzy linguistic terms to derive triangular fuzzy preference relations, and put forward additive consistency of triangular fuzzy preference relations. The main drawback is that this consistency is not robust to permutations of the names of alternative labels [50], i.e., changing the names of alternative labels influences consistency of the same triangular fuzzy judgments. This leads to our research on multiplicative consistency of TFARPRs. This paper establishes a cross-ratio-expressed triangular fuzzy multiplication based transitivity equation to define multiplicative consistency of TFARPRs, and discusses desirable properties of multiplicatively consistent TFARPRs.

Aggregation of preference information is another important issue for solving group decision making problems based on preference relations. In the process of group decision making, preference values in individual pairwise comparison matrices are usually fused into collective opinions by an aggregation operator [2,6,8,16,22,56,66]. This operator should ensure: (1) the resulting preference relation has reciprocity; (2) the resulting preference relation is consistent if all individual preference relations have consistency. Wu and Chiclana [56] proposed an approach to aggregate individual [0, 1]-valued triangular fuzzy preferences into a group one by using the triangular fuzzy weighted averaging method. This approach confirms that the resulting preference relation has reciprocity. However, the resulting preference relation is often multiplicatively inconsistent when individual preference relations are all consistent. This leads to our research on fusion of CRETFNs and aggregation of TFARPRs. This paper develops a cross-ratio-expressed triangular fuzzy weighted geometric (CRETFWG) operator to aggregate CRETFNs, and then extends the CRETFWG operator to the TFARPR weighted geometric operator for fusing TFARPRs. In addition, we devise score and uncertainty index functions for comparing and ranking CRETFNs, and present a detailed procedure for solving group decision making problems with TFARPRs.

The organization of this paper is shown as below. In the next section, we briefly review basic concepts of TFNs, TFMRPRs with multiplicative consistency and]0,1[-valued additive reciprocal preference relations. Section 3 establishes transformation methods between TFMRPRs and TFARPRs, and puts forward operations on CRETFNs. In Section 4, we introduce a multiplicative consistency definition for TFARPRs, and discuss properties of multiplicatively consistent TFARPRs. Section 5 develops some weighted geometric operators for CRETFNs and TFARPRs, and proposes a comparison approach for CRETFNs. Finally, Section 6 concludes this paper.