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From theoretical graphic objects to real free-form solids

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ABSTRACT

Formal models can be useful in computer graphics as a conceptual framework supporting representation systems. This allows to formally derive properties and algorithms and proof their correctness and validity. This paper describes a formal model based on a geometric algebra. This algebra has been used to obtain specific representation systems and study their equivalence. The representation systems derived in a natural way from this model are based on simplicial coverings and can be applied to non-manifold solids and to solids with holes. Representations have been developed for polyhedral and free-form solids. Algorithms described and proved include boolean operations and representation conversion.

The paper covers the three abstraction levels: theoretical model, representations and derived algorithms. As a practical application an experimental modeller for free-form solid has been developed (ESC-MOD system: "Extended Simplicial Chains MOdeller").

1. Introduction

Models can be defined as conceptual representations of phenomena that can be processed mathematically. These phenomena are related to the properties of the entities which we wish to model. The goal of models is to allow the study of these properties.

Computational models are based on a previously-defined mathematical abstraction that allows the correct proof of a given computational model. These abstract models are derived from the practical tests performed on the objects. Using the mathematical model it must be possible to formally characterize any of the elements studied, allowing the development of general methodologies.

The principles of geometric modeling were established by Requicha [33,34]. He defined the requirements that must be satisfied by any model of a physical object: rigidity, three dimensional homogeneity, finiteness, closed under geometric operations, finitely describable and with a well-defined boundary. These properties can be expressed using mathematical elements defined on subsets of the three-dimensional Euclidean space E^3 that are bounded, closed, regular and semi-analytic.

Working with solids implies using a computational representation system, which can be formally defined as an application between the abstract model space, M, and the representation space, R. The domain of this application must be as large as possible, and every image representation in R must be syntactically and semantically correct.

For any representation system it must be possible to compute its domain and to check the validity of all image representations; and it is also necessary to check for unambiguity and uniqueness. Besides this, it is desirable that it satisfy other properties: conciseness, easy manageability and efficiency in the sense that it allows the design of adequate algorithms.

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The verification of geometrical algorithms is possible only if we have a specification of them; that is, we must have a mathematical representation of the algorithms (for example a function with its domain and range perfectly defined).

The development of verification methods is an unsolved problem in geometric modeling. It is also necessary to develop algorithm comparison strategies for this field. Both goals require the definition of a formal framework in which it would be possible to define precisely the semantic of graphic algorithms and establish the relationship between algorithm and representation systems.

The absence of a general formal representation is solved in practice using several representations. This produces redundancy problems that can lead to inconsistency: there may be contradictions within the information held by the different representations.

One solution to this problem is to study the equivalence between representation systems. Two systems, S and S', are equivalent if for any representation, r, of S there is one representation, r' in S', that designates the same object, and vice versa. This implies that both systems must have the same domain, and that when both systems are unambiguous this condition is sufficient. At the practical level this implies ensuring the consistency between the different representations of the same object generated using different systems.

Duce [10] and Fiume [13] carried out the first work on formal models applied to computer graphics. Based on these approaches Torres and Clares [47] presented a global formalization with the goal of covering all aspects of geometric modeling. Later, Chen and Tucker [7] presented a more restricted framework.

Recently, the efforts to apply formal model to some aspects of computer graphics have increased, particularly through the development of geometric algebras. See for example the works of Vince [48], Dorst and Lasenby [9], Sang-Eon [40], Shih and Gaddipati [41] and Zaharia and Dorst [52].

A geometric algebra represents an appropriate language for the development of a valid mathematical environment. You can use it to probe fundamental algorithms in computer graphics and for optimized implementation of them (see [16,31]).

In this paper we present a unified formal framework for solid modeling from which it is possible to derive specific representation systems and algorithms in these representations. This system is defined as a universal algebra and incorporates a formal definition of the fundamental solid modeling operations. This algebra can be used to derive specific representation systems and could be used to study their equivalence. The formal nature of the model can be used to prove the correctness of the algorithms. In fact the framework has been applied to the generation of several representation systems and to the development of practical geometric algorithms.

To our knowledge this is the first work that presents a formalization for solid modeling based on geometric algebras, and develops a modeler from such formalization.

This paper focuses on the overall theory and methodology of our system. Section two explains the algebra of abstract graphic objects in the context of universal algebras; this aspect is considered as a contribution in relation to previously published works.

In section three we present a realization of the algebra based on simplicial chains applied to triangle meshes. Described is a brief summary of issues already published. In this paper we demonstrate that the model presented is a geometric algebra, that can be derived from the abstract model presented in section two. Also the content of this chapter allows the work to be self-contained.

Section 4 presents the modeler ESC-MOD as a concrete and practical application of previously introduced theoretical developments. The ESC-MOD system (Extended Simplicial Chain MODeler) is an experimental solid modeller based on the ESC representation. The high-priority objectives that have been pursued with the programming of ESC-MOD are the practical implementation of the theoretical results exposed previously in this paper, and the study of the real problems of implementation of a system with these characteristics, attempting to demonstrate the practical utility of the new representation. The system ESC-MOD is based on a robust and efficient point-in-solid test derived from ESC formulation that requires no ray-shoot calculation. Other important algorithms present in ESC-MOD are the conversion to octree and STL and direct visualization, voxelization and polygonization of free-form CSG solids.

2. Solid modeling and graphic objects algebra

This section describes the fundamental definitions of our conceptual model for graphic objects, and shows how this formalization can be used in geometric modeling. In [47] we presented a formal system that generalized the Fiume model [13] solving some of its restrictions as modeling formalism, allowing it to be applied not only to the visualization process but to the whole modeling process. Moreover, the new model includes as a particular case the Fiume formalism.

From a formal point of view a graphic object can be defined as a set of points in $\Re^n$, whose attributes can be associated to it, and satisfying properties that guarantee the validity of the representations and of the result of the operations that can be defined on them. The model must capture the object attributes that are relevant to working with its appearance and constitution, and so be able to work it. For example we can describe the material composition at each point of the object in order to be able to manage its anisotropy.

As we aim to define a general formal model, it seems appropriate to use the concept of universal algebra to define our graphic object structure. To define such algebra we consider $\Re^n$ as the support set [42].