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Competitive ratios for preemptive and non-preemptive online scheduling with nondecreasing concave machine cost ☆

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ABSTRACT

We consider an online scheduling problem where jobs arrive one by one and each job must be irrevocably scheduled on the machines. No machine is available initially. When a job arrives, we either purchase a new machine to process it or schedule it for processing on an existing machine. The objective is to minimize the sum of the makespan and the total cost of all the purchased machines. We assume that the total machine cost function is concave in the number of purchased machines. Considering both non-preemptive and preemptive variants of the problem, we prove that the competitive ratio of any non-preemptive or preemptive algorithm is at least 1.5. For the non-preemptive variant, we present an online algorithm and show that its competitive ratio is 1.6403. For the preemptive variant, we propose an online algorithm and show that its competitive ratio is 1.5654. We further prove that both competitive ratios are tight.

1. Introduction

In the classical online scheduling problem, jobs arrive one by one and each job has to be processed on a machine before the next job arrives. The number of the machines is fixed and the machines can be used to process the jobs at no cost. Naturally, it is interesting to study such a problem when machine cost is taken into consideration.

Imreh and Noga [7] first consider such an online scheduling problem with the assumption that each machine has a unit cost and no machine is initially provided. When a job arrives, either purchase a new machine to process it or schedule the job for processing on one of the purchased machines. The objective is to minimize the sum of the makespan and the cost of all the purchased machines.

For an online scheduling problem, the performance of an online algorithm A is usually measured by its *competitive ratio*, which is defined as follows:

$$c_A = \inf\left\{c | \frac{Z^A(\mathcal{J})}{Z^*(\mathcal{J})} \leqslant c \text{ for any job sequence } \mathcal{J}\right\},$$

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where $Z^A(\mathcal{J})$ (Z^A in short) denotes the objective function value produced by A and $Z^*(\mathcal{J})$ (Z^* in short) denotes the optimal objective function value of its offline counterpart. An algorithm with a competitive ratio at most c is called a c-competitive algorithm. An online scheduling problem has a *lower bound* ρ if no online algorithm has a competitive ratio smaller than or equal to ρ. An online algorithm is said to be *optimal* if its competitive ratio matches the lower bound for the online problem.

Imreh and Noga [7] consider two online models, namely the *List Model*, in which the jobs arrive one by one, and the *Time Model*, in which the jobs arrive over time. They present a 1.618-competitive algorithm for the List Model and a 1.693-competitive algorithm for the Time Model. Furthermore, they prove that the lower bounds for the two models are at least 4/3 and 1.186, respectively. Seiden [12] provides a randomized lower bound of $6e/(6e-1) > 1.06532$. Many subsequent papers consider the List Model. Dósa and He [2] present an improved online algorithm with a competitive ratio of at most $(2\sqrt{6}+3)/5 \approx 1.5798$. Recently, Dósa and Tan [4] make a further improvement by presenting an algorithm with a competitive ratio of $(2+\sqrt{7})/3 \approx 1.5486$ and giving a new lower bound of $\sqrt{2}$. The main idea of the machine purchasing strategy in these two papers is to consider the current number of purchased machines and the current makespan, while [7] only considers the total size of the newly arrived jobs in deciding whether or not a new machine should be purchased. In addition, some semi-online variants are considered in [1,2,5,8,9]. Jiang and He [8] consider the preemptive version of the List Model and present an online algorithm with a competitive ratio of $(2\sqrt{6}+2)/5 \approx 1.3798$. Dósa and He [3] and Nagy-György and Imreh [10] consider another extension of the problem in which new machines may be purchased or the jobs may be rejected.

All the above results are based on the assumption that each machine has a unit cost, which however is not very realistic since in practice the price of each machine may be different. Thus, Imreh [6] considers the problem with a general cost function. He assumes that the cost of each machine is arbitrary and the machine cost function $f(m)$ is a nondecreasing function of the number of purchased machines m. That is, the value $f(m) - f(m-1)$ is the cost of the mth purchased machine. In Comparison, the machine cost function in [7] is an identity function, i.e., $f(m) = m$. For the general cost function, Imreh [6] presents a $(3+\sqrt{5})/2 \approx 2.618$-competitive algorithm and shows that a lower bound is at least 2. For the special case where the size of each job is not larger than the minimum cost of the machines, he presents an optimal algorithm with a competitive ratio of 2. He also considers a more general version where the available machines have different speeds.

Recently, Ruiz-Torres et al. [11] consider the tradeoffs between regular measures of performance and machine cost for the identical parallel-machine problem. They propose that the cost of each machine is a function of the load of the machine. They study the case where the function is concave, i.e., as the load increases, the average cost per unit decreases.

In this paper we study the problem under the assumption that the total machine cost function, denoted by $f(m)$, is concave in the number of purchased machines m. Our study has the following advantages over the exiting works: (a) The case we study is more general than the identity machine cost function. It is easy to see that if $f(m) = m$, i.e., each machine has a unit cost, then our problem becomes the same as those considered in [2,4,7,8]. (b) The concave cost function is more realistic in practice. Intuitively, the more products a customer purchases, the cheaper the price becomes, which is a property of the concave function. In other words, the average cost of the machines decreases with increasing number of purchased machines, i.e., $\triangle f(m) = f(m) - f(m-1)$ deceases in m while $f(m)$ increases in m.

Our contributions are as follows: We first give a useful property of the concave function and a lower bound on the optimal objective function value. We show that the lower bound is at least 1.5 regardless of whether or not preemption is allowed. We then present efficient algorithms for both non-preemptive and preemptive variants of the problem. For the non-preemptive variant, we present an online algorithm with a competitive ratio $(9+\sqrt{17})/8 \approx 1.6403$. For the preemptive variant, we propose an online algorithm with a competitive ratio 1.5654. Moreover, we prove that both bounds are tight, i.e., our algorithms provide the best possible bounds that are sufficiently close to 1.5, the known lower bound.

Table 1 summarizes our results and the major related works. As shown in the table, our problem is more general than the existing ones with identity cost function, while the competitive ratio of our problem (1.6403) is not very much greater than the best known result (1.5486) [4]. Moreover, we obtain that the gap between the competitive ratio and lower bound is $1.6403 - 1.5 = 0.0903$, which is smaller than the gap $1.5486 - 1.4142 = 0.1344$ reported in [4]. While our problem is a special case of the problem considered by Imreh [6], we greatly improve the result from 2.618 to 1.6403. For the preemptive variant of the problem, the competitive ratio of our algorithm is very close to the lower bound, so it can almost be regarded as an optimal online algorithm.

The reason why our algorithms have good performance is because we apply better strategies to decide whether or not to purchase new machines. In our algorithms, once a job arrives, we take into consideration the size of the current job, the size

Table 1
Results for online scheduling with machine cost.

Cost function	(Non-)preemptive	Competitive ratio of online algorithm	Lower bound	Reference
Identity function	Non-preemptive	1.618	4/3	Imreh and Noga [7]
Identity function	Non-preemptive	1.5798		Dósa and He [2]
Identity function	Non-preemptive	1.5486	$\sqrt{2}$	Dósa and Tan [4]
Identity function	Preemptive	1.3798	4/3	Jiang and He [8]
Concave function	Non-preemptive	1.6403	3/2	This paper
Concave function	Preemptive	1.5654	3/2	This paper
General function	Non-preemptive	2.618	2	Imreh [6]