Information Sciences 269 (2014) 286–299

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Dominating problems in swapped networks

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ARTICLE INFO

Article history:
Received 25 April 2012
Received in revised form 25 August 2013
Accepted 19 October 2013
Available online 25 October 2013

Keywords:
Swapped network
Dominating set
Connected dominating set
$\mathcal{NP}$-hard
Approximation

ABSTRACT

Swapped Networks (SNs) are a family of two-level interconnection networks, suitable for constructing large parallel and distributed systems. In this paper, the Minimum Dominating Set (MDS) problem and the Minimum Connected Dominating Set (MCDS) problem in SNs are investigated based on the connectivity rule of SNs. We prove the two problems in SNs are $\mathcal{NP}$-hard, and present two efficient algorithms for building dominating sets and connected dominating sets in SNs. The proposed algorithms use as input a given (connected) dominating set of the factor network, and yield a good approximation of an MDS or MCDS for the SN provided that the input is a good approximation of an MDS or MCDS for the factor network. We also derive several non-trivial bounds on the (connected) domination parameters of SNs. We believe this work is of theoretical interest in graph theory since SNs form a family of graphs. It may also motivate further research on dominating problems in SNs with their potential applications.

1. Introduction

A fundamental issue in various network systems is how to efficiently control, track, or detect behaviors of the constituent nodes in the network, and a solution often involves building a dominating set [31]. Since the concept of domination set was first proposed in 1960s [20], many variants of dominating set have arisen from different application areas, including communication equipment layout, city route plan, numerical analysis, VLSI design, information search, CPU array layout, communication scheme in supercomputers and network controllability [12,24,33,36]. Among all the variants, Dominating Set (DS) and Connected Dominating Set (CDS) are the two basic versions. For a graph $G = (V, E)$, a DS is a node set $X \subseteq V$ such that every node not in X is adjacent to at least one node in X and a CDS is a DS whose induced subgraph is connected.

Clearly, any DS or CDS of a network enjoys the property that the processors within such a set are capable of delivering messages to all the other processors in a single step. Therefore, DSs and CDSs are widely used to create effective layouts for servers or common resources in networks and design efficient routing algorithms [7,25,27]. The *domination number* $\gamma(G)$ is the size of a Minimum DS (MDS) of graph G, and the *connected domination number* $\gamma_c(G)$ is the size of a Minimum CDS (MCDS) of graph G. These domination parameters can be used to measure the controllability of a network [19]. For example, Cooper et al. studied the domination numbers of several kinds of web networks and found these domination numbers are fairly large [4], which implies that a crawler that wants to use a DS to explore such a web network may have to store a large proportion of nodes in the network. It is well known that for a general graph G, finding an MDS or MCDS, even only computing $\gamma(G)$ or $\gamma_c(G)$, is $\mathcal{NP}$-hard [9,11]. In the literature many efforts have been made to obtain these domination parameters for some specific networks, such as grid graphs[10] and butterfly graphs[13], or establish non-trivial bounds on these domination parameters for various types of networks, such as hypercubes[1], de Bruijn graphs [30] and cartesian product

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0020-0255/$ - see front matter
http://dx.doi.org/10.1016/j.ins.2013.10.020

graphs [34]. In an application system, it is also important to develop efficient algorithms to find an MDS or MCDS for the underlying network. For example in the context of optimal sensor placement, if each sensor can eavesdrop on its neighbors, then an MDS will give a minimum cost placement provided that placing a sensor incurs a non-zero cost. In the context of a mobile ad hoc network, MCDSs are always used as backbones to improve the network performance, optimize the routing protocols, and increase the lifetime of the network [5]. Many efficient algorithms have been developed for finding a good approximation of an MDS [2,18] or a good approximation of an MCDS [14,26,32] based on some specific features of the application networks.

In this paper, we investigate the MDS problem and MCDS problem in Swapped Networks (SNs) that are suitable for building parallel and distributed systems [17,21]. Specifically, given a n-node factor network, the corresponding SN is a two-level architecture composed of n copies of the factor network. Each copy is served as a cluster and all the clusters are connected using a simple inter-cluster connectivity rule, which leads to the regularity, modularity, fault tolerance, and algorithmic efficiency [3,6,16]. SNs have received considerable attentions over the past two decades because the architecture is shown to outperform some well known architectures in many aspects such as the maximum node degree, scalable cost and flexibility.

A number of algorithms have been developed for routing, sorting and numerical analysis in SNs [22,28,35]. In addition, since in parallel and distributed systems, we have to broadcast messages in many occasions, such as clock synchronizing, load sharing and replica updating, the MDS and MCDS problems are of practical interest in SNs. Recently, Mahafzah et al. conducted a experimental study on DSs in SNs built from mesh factor networks [15]. In their study, DSs were used to design collective communication operations in these SNs, and the advantages of DS approach-based collective communication operations over other approaches were confirmed by simulation analysis. To the best of our knowledge, no attempt so far has been made to characterize the graph-theoretic structures of MDSs and MCDSs in SNs. In this study, we investigate MDSs and MCDSs in SNs both practically and theoretically. We establish the $\mathcal{NP}$-hardness of finding an MDS or MCDS in SNs built from arbitrary factor networks. We also derive several non-trivial bounds on the (connected) domination parameters of SNs. Given a DS or CDS in a factor network, we develop efficient algorithms for fast building a small DS or CDS in the corresponding SN. Our algorithms are only based on the given DS or CDS, and thus are applicable to all types of SNs.

The remainder of this paper is organized as follows. In Section 2, we introduce the notations and definitions. In Section 3, we prove the $\mathcal{NP}$-hardness of the MDS problem and the MCDS problem in SNs. In Section 4, we present two approximation algorithms for these two problems, and obtain several lower bounds and upper bounds on the associated domination parameters. In Section 5, we give the simulation results and the analysis. In Section 6, we conclude the paper.

2. Notations and definitions

We begin by describing some notations and definitions used in this paper. Let $G = (V_G, E_G)$ be a graph with node set V_G and edge set E_G. For node $v \in V_G$, we denote by $N_G(v)$ the set of neighbor nodes of v, and let $N_G[v] = \{v\} \cup N_G(v)$. For a subset of nodes $D \subseteq V_G$, we denote by $N_G(D)$ the set of neighbors for all the nodes in D, and let $N_G[D] = D \cup N_G(D)$. For node $v \in V_G$, we denote by $deg_G(v) = |N_G(v)|$ the degree of node v. The maximum degree and the minimum degree among the nodes of G are denoted by $\Delta(G)$ and $\delta(G)$ respectively. For other notations we follow [29]. The definition of SN is as follows.

Definition 1 (*SN 21*). Given a factor network $G = (V_G, E_G)$, the swapped network built from graph G, denoted by $SN(G)$, is a graph with node set $V_{SN(G)} = \{g_p | g, p \in V_G\}$ and edge set $E_{SN(G)} = \{(g_{p_1}, g_{p_2}) | g, p_1, p_2 \in V_G, (p_1, p_2) \in E_G\} \bigcup \{(g_p, p_g) | g, p \in V_G, g \neq p\}$.

Let $|V_G| = n$ and $V_G = \{1, 2, \ldots, n\}$. By Definition 1, $SN(G)$ consists of n subnetworks (called clusters $1, 2, \ldots, n$, respectively), each of which is isomorphic to factor network G. A node with identifier g_p $(g, p \in \{1, 2, \ldots, n\})$ corresponds to node p of cluster g, where g is called its cluster label and p is called its node label. Fig. 1(a) and (b) give two example factor networks. The corresponding swapped networks are respectively shown in Fig. 2(a) and (b). For each node g_p, we call it an inner node for cluster g, and call it an outer node for any other cluster $g\prime$. For example, in Fig. 2(a), node 1_1 is an inner node of cluster 1, and it is an outer node for clusters 2, 3 and 4. For each edge (g_{p_1}, g_{p_2}), we call it an inner edge in cluster g. For

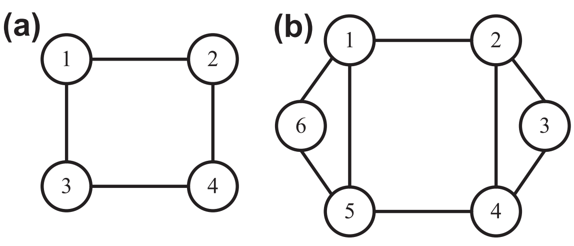

Fig. 1. Two example factor networks.