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A goal programming model of obtaining the priority weights from an interval preference relation

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ABSTRACT

The main aim of this paper is to investigate the weight vectors of interval fuzzy preference relations and interval multiplicative preference relations. In view of the excellent properties of multiplicative transitivity in modeling the consistency of fuzzy preference relations, based on the multiplicative consistency property, a goal programming model of obtaining the priority weights from an interval fuzzy preference relation is introduced along with some of its desired properties. The priority vector of a fuzzy preference relation can also be derived using this goal programming model, where the optimal value of the objective function is always equal to zero. Numerical examples are provided to illustrate the validity of the proposed model.

1. Introduction

Because pairwise comparison methods have an incomparable advantage over non-pairwise methods, decision making based on preference relations has gained widespread attention over the past few decades [1–3,6,7,9,10–14,18,20–22,25–28,36,37,46]. Comparatively speaking, fuzzy preference relations and multiplicative preference relations are the two most common forms that are used to express preference information in decision making. In preference relations-based decision making, consistency and the weight vector are two important topics that arouse great interest from decision makers (DMs). Consistency measures the level of agreement among the preference values provided by the individual DMs [44]. The lack of consistency in decision making with preference relations generally results in inconsistent conclusions. Many methods regarding consistency measures and improvements of preference relations have been proposed. The weight priority of preference relation, which represents the weights of criteria or the ranking of alternatives, plays a key role in decision making.

Due to the uncertainties inherent in the real world and the limitations of cognitive abilities, DMs sometimes have difficulty expressing the inputs of preference relations as crisp values. In such cases, interval preference relations, such as interval multiplicative preference relations and interval fuzzy preference relations, may serve as suitable tools to better represent the DMs' opinions. As a result, deriving reasonable weight vectors for interval preference relations is becoming a hot topic. Most of the related literature takes the elements of the weight vector of an interval preference relation as intervals. For example, Xu and Chen [41] detailed the concepts of additive consistent interval fuzzy preference relations and multiplicative consistent interval fuzzy preference relations and established linear programming models for deriving the priority weights from various interval fuzzy preference relations. Wang and Elhag [33] proposed a goal programming method for obtaining interval weights from an interval comparison matrix regardless of whether it was consistent or inconsistent. Zhang

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et al. [45] proposed a method for deriving priority weights by directly extracting the consistent numerical valued additive-based pairwise comparison matrix. Liu et al. [24] provided a definition of interval consistent fuzzy preference and proposed a method for generating the interval priority weights from interval fuzzy preference relations using the convex combination method. Wang and Li [34] developed goal programming models to derive interval weights from interval fuzzy preference relations by analyzing the relationships between interval weights and consistent interval fuzzy preference relations. Sugihara et al. [31] proposed an interval approach for obtaining the interval weights of priorities where the upper and lower models are used to approximate the interval comparison matrix. GenÇ et al. [19] proposed an approach for checking whether an interval fuzzy preference relation is consistent and derived a formula to determine the priority vector of a consistent interval fuzzy preference relation, which produces the same results as those of the linear programming models proposed by Xu and Chen [41]. Chen and Xu [5] discussed the interval reciprocal comparison matrices and suggested a new fuzzy programming method to derive the priority vector from an interval reciprocal comparison matrix. Wang and Chen [35] developed a logarithmic least squares model to derive interval weights from any interval fuzzy preference relation based on geometric transitivity.

The above literature review shows that mathematical programming models, including goal programming and linear programming, are widely used to derive the weight prioritys of interval preference relations. Some of the methods mentioned above are unsuitable for dealing with crisp preference relations or inconsistent interval preference relations. Some methods are somewhat complicated: more than one mathematical programming model should be constructed to derive the weight vectors of inconsistent interval preference relations or the constructed models should consider each preference value individually. Because multiplicative transitivity has the most appropriate property to model the cardinal consistency of reciprocal preference relations [8], based on the multiplicative consistency property, this paper proposes a goal programming model to derive the interval weight vector of an interval fuzzy preference relation. The examples provided illustrate the advantages of the proposed method over some existing similar methods: the weight vector preserves more original information about an interval preference relation, and the weight vector is not inclined to influence outliers or possible errors among the inputs of preference relations.

To accomplish this goal, the rest of this paper is structured as follows. In Section 2, a brief introduction to the basic notions is provided. Section 3 proposes a goal programming model to generate the interval weight vector of an interval preference relation. In Section 4, a discussion and comparison with other similar methods are provided. Section 5 extends the goal programming model proposed in Section 3 to solve group decision-making problems. Section 6 provides the conclusions.

2. Preliminaries

For simplicity, let $X = (x_1, x_2, \dots, x_n)$ be a finite set of alternatives and $I = \{1, 2\dots, n\}$ be the set of the index.

Definition 1 [29]**.** A multiplicative preference relation A on a set of alternatives X is represented by a reciprocal matrix $A = (a_{ij})_{n\times n}$ with

$$a_{ij} > 0, \quad a_{ij}a_{ji} = 1, \quad a_{ii} = 1, \quad \forall i, j \in I \tag{1}$$

where a_{ij} expresses the ratio of the preference intensity of alternative x_i to that of x_j. Specifically, $a_{ij} < 1$ indicates that x_j is preferred to x_i; $a_{ij} = 1$ indicates indifference between x_i and x_j; and $a_{ij} > 1$ indicates that x_i is preferred to x_j.

Definition 2 [30]**.** A multiplicative preference relation $A = (a_{ij})_{n\times n}$ is consistent if the following transitivity is satisfied:

$$a_{ij} = a_{il}a_{lj}, \quad \forall i, j, l \in I \tag{2}$$

Definition 3 [4,15–17]**.** A fuzzy preference relation R on the set X is represented by a complementary matrix $R = (r_{ij})_{n\times n} \subset X \times X$ with

$$1 \ge r_{ij} \ge 0, \quad r_{ij} + r_{ji} = 1, \quad r_{ii} = 0.5, \quad \forall i, j \in I \tag{3}$$

where r_{ij} represents a crisp preference degree of alternative x_i over x_j. Specifically, $r_{ij} = 0$ indicates that x_j is absolutely preferred to x_i; $r_{ij} = 0.5$ indicates indifference between x_i and x_j; $r_{ij} > 0.5$ indicates that x_i is preferred to x_j; and $r_{ij} = 1$ indicates that x_i is absolutely preferred to x_j.

Tanino [32] defined the multiplicative consistent fuzzy preference relation.

Definition 4 [32]**.** A fuzzy preference relation $R = (r_{ij})_{n\times n}$ is multiplicatively consistent if and only if the following condition of transitivity holds:

$$\frac{r_{ji}r_{kj}}{r_{ij}r_{jk}} = \frac{r_{ki}}{r_{ik}} \tag{4}$$

where $1 > r_{ij} > 0$, $\forall i, j \in I$.

Note that Definition 4 is given based on the assumptions: $r_{ij} \ne 0$ and $r_{ij} \ne 1$, $\forall i, j \in I$, which are also taken in the following sections.