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A comparative study of type-1 fuzzy logic systems, interval type-2 fuzzy logic systems and generalized type-2 fuzzy logic systems in control problems

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ABSTRACT

This paper presents a comparative study of type-2 fuzzy logic systems with respect to interval type-2 and type-1 fuzzy logic systems to show the efficiency and performance of a generalized type-2 fuzzy logic controller (GT2FLC). We used different types of fuzzy logic systems for designing the fuzzy controllers of complex non-linear plants. The theory of alpha planes is used for approximating generalized type-2 fuzzy logic in fuzzy controllers. In the defuzzification process, the Karnik and Mendel Algorithm is used. Simulation results with a type-1 fuzzy logic controller (T1FLC), an interval type-2 fuzzy logic controller (IT2FLC) and with a generalized type-2 fuzzy logic controller (GT2FLC) for benchmark plants are presented. The advantage of using generalized type-2 fuzzy logic in fuzzy controllers is verified with four benchmark problems. We considered different levels of noise, number of alpha planes and four types of membership functions in the simulations for comparison and to analyze the approach of generalized type-2 fuzzy logic systems when applied in fuzzy control.

1. Introduction

Fuzzy control systems combine information of human experts (natural language) with measurements and mathematical models. Fuzzy systems transform the knowledge base into a mathematical formulation that has proven to be very efficient in many applications [1,4,5,6,14,45,48].

Type-2 fuzzy models have emerged as an interesting generalization of type-1 fuzzy models based upon fuzzy sets. In fact, these models are also called generalized type-2 fuzzy models. There have been a number of claims put forward as to the relevance of type-2 fuzzy sets being regarded as generic building constructs of fuzzy models. Fuzzy controllers have the advantage that they can be adaptive when disturbances in the plant are present. The main advantage of using generalized type-2 fuzzy control (GT2FLC) is that the plants show a higher degree of stability in the simulations. When we consider noise presence for the GT2FLC, the results show that the Generalized Type-2 Fuzzy Logic System has better stability characteristics.

This paper considers several experiments in the simulation of four benchmark control problems with a type-1 fuzzy logic controller (T1FLC) and with an interval type-2 fuzzy logic controller (IT2FLC), where the authors conclude that the IT2FLC is

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better than T1FLC based on the simulation error minimization. We also realized the comparison with the three methods to observe the behavior and the improvement that a GT2FLC can offer with respect to T1FLC and IT2FLC.

Few works can be found in the literature related to the implementation of generalized type-2 fuzzy logic systems in the area of fuzzy control. It is because of this fact that we use the concepts of the $\alpha-$ planes representation and generalized type-2 fuzzy sets to improve the design of fuzzy logic systems and allow the analysis of benchmark problems more efficiently with high levels noise in the model.

In the last five years, there has been a significant increase of the research on higher order forms of fuzzy logic systems, in particular the use of interval type-2 fuzzy logic [1,2,11,12,33,44,51,52]. In addition with the advancement of IT2FLS, uncertainty could finally be directly incorporated into the Fuzzy Sets. Although the boom of research with IT2FLS is recent, there is still much to be explored, and some current examples of research are shown by, a fuzzy model of computing with words [11], fuzzy operations [12], a simplified IT2FLS [22], type-reduction algorithms [30], the centroid of triangular and Gaussian IT2FSs [38], and enhanced type-reduction [40]. More recently generalized type-2 fuzzy logic systems have been proposed; of course, the idea of going into higher orders or types of fuzzy logic is to construct better models of uncertainty. In this sense, it is theoretically expected that generalized type-2 fuzzy logic will allow better management of uncertainty [49,50]. However, generalized type-2 requires a higher computational overhead and several efforts have been put forward in order to limit the complexity of generalized type-2 fuzzy logic systems; for example, Wagner and Hagras [3],[41–43] have introduced the zSlices-based representation, and Mendel and Liu [21,22], have put forward a representation based on α-planes, which both enable the representation of, and computation with, generalized type-2 fuzzy sets [22,23]. And as such, there are also very few application research works where GT2FS are used, for example, in a GT2FS based on face-space approach to emotion recognition [8], a general type-2 fuzzy inference systems; analysis, design and computational aspects [16], fuzzy c-means for uncertain fuzzy clustering [17], monotone centroid flow algorithm for type-reduction [18], multi-criteria group decision making [19], edge detection for image processing [20], matching GT2FS by comparing the vertical slices [34], generalized type-2 fuzzy systems with interval type-2 and type-1 fuzzy systems applied to fuzzy control [35], the information granule numerical evidence [50], computing with words for discrete GT2FS [51]. In addition in the field of fuzzy control, a comparison of fuzzy controllers for the Water Tank with Type-1 and Type-2 Fuzzy Logic is presented in [1], and the improvement of GT2FLS over IT2FLS and T1FLS for controlling a mobile robot is presented in [35]. In this paper, we use the α-plane representation, which enables the representation of and computation with generalized type-2 fuzzy sets.

The main contribution of this paper is a comparative study based on generalized type-2 fuzzy logic for the design and implementation of fuzzy controllers, which allows for better modeling of the uncertainty that exists in achieving control of non-linear plants. Also, the comparative study of T1FLC, IT2FLC and GT2FLC as tools for modeling complex problems in control.

Based on the literature and experience in the area of fuzzy control, we expect that GT2FLC can demonstrate that the stabilization and the performance of the simulations are improved, especially when noise levels are considered in the model, and better results are obtained when compared with respect to type-1 FLC and interval type-2 FLC.

The rest of the paper is organized as follows. Section 2 describes some basic concepts of fuzzy logic systems; type-1 FLS, interval type-2 FLS, generalized type-2 fuzzy logic systems and the theory of alpha planes. Section 3 shows the benchmark problems that we consider in the simulations. Section 4 shows the simulation results with the implemented generalized type-2 fuzzy logic controller and a comparison with the type-1 and interval type-2 fuzzy logic controllers, so that the advantage of using a generalized type-2 fuzzy logic controller is fully appreciated. Finally, Section 5 offers some conclusions of this work.

2. Basic concepts of fuzzy logic systems

2.1. Definition of type-1 fuzzy logic systems

A type-1 fuzzy set in the universe X is characterized by a membership function $u_A(x)$ taking values on the interval [0,1] and can be represented as a set of ordered pairs of an element and the membership degree of an element to the set and are defined by the following Eq. (1) [45–48]:

$$A = \{(x, \mu_A(x))|x \in X\} \tag{1}$$

Where $\mu_A : X \to [0, 1]$.

In this definition $\mu_A(x)$ represents the membership degree of the element $x \in X$ to the set *A*. In this work we are going the use the following notation: $A(x) = \mu_A(x)$ for all $x \in X$.

2.2. Definition of Interval type-2 fuzzy logic systems

Based on Zadeh's ideas, in 1979 Mizumoto and Tanaka [3] presented the mathematical definition of a type-2 fuzzy set. Since then, several authors have studied these sets, in [25–27] Mendel, John and Mouzouris defined these sets as follows [3].