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A fuzzy envelope for hesitant fuzzy linguistic term set and its application to multicriteria decision making

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ABSTRACT

Decision making is a process common to human beings. The uncertainty and fuzziness of problems demand the use of the fuzzy linguistic approach to model qualitative aspects of problems related to decision. The recent proposal of hesitant fuzzy linguistic term sets supports the elicitation of comparative linguistic expressions in hesitant situations when experts hesitate among different linguistic terms to provide their assessments. The use of linguistic intervals whose results lose their initial fuzzy representation was introduced to facilitate the computing processes in which such expressions are used. The aim of this paper is to present a new representation of the hesitant fuzzy linguistic term sets by means of a fuzzy envelope to carry out the computing with words processes. This new fuzzy envelope can be directly applied to fuzzy multicriteria decision making models. An illustrative example of its application to a supplier selection problem through the use of fuzzy TOPSIS is presented.

1. Introduction

Decision making is a universal process in the life of human beings, which can be described as the final outcome of some mental and reasoning processes that lead to the selection of the best alternative or set of alternatives. Decision making problems [21] are usually defined in uncertain and imprecise situations. In such cases, it is appropriate for experts to provide their preferences or assessments using linguistic information rather than quantitative values. This has led to the use of different approaches, such as fuzzy logic [38] and the fuzzy linguistic approach [39], to model this type of uncertainty and vagueness in decision making problems. The use of linguistic information implies the need for computing with words (CWW) processes [12,17,41] that can be carried out by different linguistic computational models [12,17]. These models follow the computational scheme depicted in Fig. 1, in which Yager [37] highlights the translation and retranslation phases in the CWW processes. The former involves taking linguistic information and translating it into a machine manipulative format, and the latter consists of taking the results from the machine manipulative format and transforming them into linguistic information to facilitate their being understood by human beings, which is one of the main objectives of CWW [18].

The complexity of real world decision problems is often caused by uncertainty regarding the alternatives. The use of linguistic information has provided successful results for managing this. However, it is sometimes limited by the fact that the linguistic models use only one linguistic term, which may not reflect exactly what the experts mean. Usually, in decision problems defined in a linguistic context with a high degree of uncertainty, experts might hesitate among different linguistic terms and need richer linguistic expressions to express their assessments. Different linguistic proposals have been

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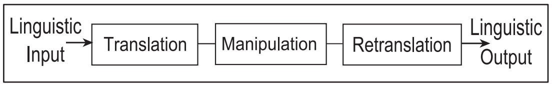

Fig. 1. Computing with words scheme.

introduced in the literature to provide richer linguistic expressions than single linguistic terms. Wang and Hao [29] proposed the use of proportional 2-tuple based on the proportion of two consecutive linguistic terms. Ma et al. [16] presented a linguistic model to increase the flexibility of the linguistic expressions, merging different single linguistic terms into a new synthesized term. Tang and Zheng [26] introduced another linguistic model to manage linguistic expressions built by logical connectives. Nevertheless, these proposals generate expressions far away from the natural language used by experts in decision problems or else they do not have any defined formalization.

A recent proposal was introduced by Rodríguez et al. [23] to improve the elicitation of linguistic information in decision making by using hesitant fuzzy linguistic term sets (HFLTS) when experts hesitate among several linguistic terms to express their assessments. This approach provides experts with greater flexibility to elicit comparative linguistic expressions close to human beings' cognitive model by using context-free grammars that formalize the generation of flexible linguistic expressions.

The use of comparative linguistic expressions based on context-free grammars and HFLTS has been applied to different decision making problems [23,24] in which the computational linguistic model deals with linguistic intervals obtained by the envelope of HFLTS [23] and operates on them with a symbolic model that finally obtains crisp values, losing the initial fuzzy representation. Keeping in mind the fuzzy linguistic approach in which the linguistic terms are represented by a syntax and fuzzy semantics, it seems reasonable that the semantics of the comparative linguistic expressions based on a context-free grammar and HFLTS should be represented by fuzzy membership functions that model the uncertainty and vagueness expressed by such comparative linguistic expressions.

The aim of this paper is to introduce a fuzzy representation for comparative linguistic expressions that will be based on a new fuzzy envelope for HFLTS that will represent the expressions through a fuzzy membership function obtained from the multiple linguistic terms that compound the HFLTS, and aggregated using the OWA operator [33]. Such a fuzzy representation will facilitate the CWW processes in fuzzy multicriteria decision making models [13,19] that deal with HFLTS. To show the performance of the proposed fuzzy envelope, a supplier selection multicriteria decision making problem is presented and solved by a fuzzy TOPSIS model [2,5,30] dealing with comparative linguistic expressions.

The remainder of the paper is structured as follows: Section 2 reviews the fuzzy linguistic approach basis of the HFLTS, the elicitation of comparative linguistic expressions based on context-free grammars and HFLTS, and the OWA operator used to compute the novel fuzzy envelope. Section 3 proposes a fuzzy envelope for HFLTS based on fuzzy membership functions. Section 4 shows the application of the fuzzy envelope in a supplier selection multicriteria decision making problem. And finally, Section 5 makes some concluding remarks.

2. Preliminaries

This section reviews the fuzzy linguistic approach basis of the HFLTS, the elicitation of comparative linguistic expressions and the OWA operator used to obtain the proposed fuzzy envelope for HFLTS.

2.1. Fuzzy linguistic approach

In many real decision making situations the use of linguistic information rather than numerical information is straightforward due to the imprecise framework in which such problems are defined. In such situations, the fuzzy linguistic approach [39] represents the linguistic information by means of linguistic variables.

Zadeh introduced the concept of the "linguistic variable" as *a variable whose values are not numbers but words or sentences in a natural or artificial language*. It is not as precise as a number but it is closer to human beings' cognitive processes. It is defined as follows:

Definition 1. [39] A linguistic variable is characterized by a quintuple $(H, T(H), U, G, M)$ in which H is the name of the variable; $T(H)$ is the term set of H, i.e., the collection of its linguistic values; U is a universe of discourse; G is a syntactic rule which generates the terms in $T(H)$; and M is a semantic rule which associates with each linguistic value X its meaning, $M(X)$ denotes a fuzzy subset of U.

To deal with linguistic variables, it is necessary to choose appropriate linguistic descriptors of the linguistic terms and their semantics. There are different approaches [22] to such selection. To choose the linguistic descriptors we will use an approach that consists of directly applying the term set by considering all the terms distributed on a scale that has a defined order [36]. In such cases, it is required that a linguistic term set $S = \{s_0, s_1, \ldots, s_g\}$ satisfies the following conditions:

1. An order of the terms of S: $s_i \leqslant s_j$ iff $i \leqslant j$;