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An ordinal approach to computing with words and the preference–aversion model

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ABSTRACT

Computing with words (CWW) explores the brain's ability to handle and evaluate perceptions through language, i.e., by means of the linguistic representation of information and knowledge. On the other hand, standard preference structures examine decision problems through the decomposition of the preference predicate into the simpler situations of strict preference, indifference and incomparability. Hence, following the distinctive cognitive/neurological features for perceiving positive and negative stimuli in separate regions of the brain, we consider two separate and opposite poles of preference and aversion, and obtain an extended preference structure named the Preference–aversion (P–A) structure. In this way, examining the meaning of words under an ordinal scale and using CWW's methodology, we are able to formulate the P–A model under a simple and purely linguistic approach to decision making, obtaining a solution based on the preference and non-aversion order.

1. Introduction

The preference predicate is commonly represented by means of binary relations, which are decomposed into different types of decision situations. Such situations can make reference to strict preference, identifying one desired solution, or to other type of situations, like e.g., indifference, incomparability or ignorance, expressing some kind of *decision neutrality*. In this way, the meaning of preference rests on the human conditions of subjectivity which characterize intelligence and rationality. Therefore, the decision process of the individual can be better described and understood taking into account the linguistic and cognitive attributes of human thinking.

Different models exist for the linguistic treatment of information (see, e.g., [13,34,43]). One of them is *Computing with Words*, CWW (see, e.g., [15,16,24]). The CWW methodology makes use of *linguistic variables* [43], such that their values are not numbers but words. Hence, it is possible to work on purely linguistic terms under an analytical and mathematical framework. On the other hand, the cognitive/epistemic states of a decision problem can be explored attending to the brain's capability for distinguishing between positive and negative stimuli and perceptions (see e.g., [20,28]). In this way, two separate evaluations are used to measure the positive and the negative attributes on a given set of alternatives A, following the Preference–aversion (P–A) model [11,12].

Under the CWW methodology for linguistic decision analysis, two prior steps are necessary to be carried out before aggregating the information and exploiting it in order to obtain a solution [16]. First, the granularity of the linguistic term set has to be determined along with its labels and semantics, establishing an expression domain that provides the linguistic performance values of the alternatives. Second, aggregation operators have to be chosen for combining the linguistic

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performance values. Therefore, following the P–A model, the performance of the alternatives is valued separately regarding their positive and negative dimensions, and only then, the linguistic evaluations can be aggregated in order to identify the most suitable alternatives for solving the decision problem.

The objectives of this paper are to present the P–A model under a purely linguistic approach, exploring preference semantics and the meaning of words and predicates, and to set the methodology for obtaining a solution based on the preference and non-aversion order. For doing so, this paper is organized as follows. In Section 2 we give the mathematical formulation of the P–A model under an ordinal linguistic evaluation of the alternatives regarding the preference predicate. Then, in Section 3, we explore preference semantics and exploit the linguistic information contained in the P–A structure. This exploitation phase is developed by means of the self-control principle and the preference and non-aversion order. Finally, we end with some final remarks.

2. Computing with words and the preference–aversion model

In this section we review the CWW methodology for dealing with linguistic information [25,43], and apply it on the P–A model [12].

2.1. Linguistic evaluation of preference

The preference predicate "*is at least as desired as*", represented by R, is commonly understood by means of preference structures (see e.g., [33]). A preference structure, defined on a set of alternatives A, decomposes the meaning of R into three basic binary relations (P,I,J), and describes the epistemic states of the individual regarding the problem of deciding among the alternatives in A. Hence, for any pair $(a,b) \in A \times A$, the decision problem agrees with *strict preference* (P), if the pair of alternatives (a,b) verify the predicate "*a is more desired than b*"; *indifference* (I), if the pair of alternatives (a,b) verify the predicate "*a is as desired as b*"; or *incomparability* (J), if the pair of alternatives (a,b) verify the predicate "*a is incomparable with b*".

Under this standard approach, the binary relation $R = P \cup I$ is commonly referred to as the *large/weak preference relation* of the structure $\langle P,I,J\rangle$ and R^{-1} stands for the inverse of R, such that $R^{-1}(a,b) = R(b,a)$. Besides, the relations (P,I,J) are assumed to satisfy some additional conditions: I and J are symmetrical, I is reflexive, J irreflexive, P asymmetrical, and

$$P \cap I = \emptyset, \tag{1}$$

$$P \cap J = \emptyset, \tag{2}$$

$$J \cap I = \emptyset, \tag{3}$$

$$P \cup P^{-1} \cup I \cup J = A \times A. \tag{4}$$

From (1)–(3) we have that one and only one of the binary relations of the structure $\langle P,I,J\rangle$ hold, while condition (4) assumes that any decision problem can be properly described by anyone of its four components.

In this way, it is assumed that the individual understands his decision by just one situation of $\langle P,I,J\rangle$, where there are two distinct states of *decision neutrality*. The first state is indifference I, where no alternative is clearly better than the other one and no *best* decision is available. The second state is incomparability J, where alternatives cannot be compared, and once again, no *winner* can be identified.

It is well known that using common fuzzy operators, these structures can be extended to a fuzzy setting, where the verification of the properties of preference can be verified up to a degree of intensity (see, e.g., [10,26,40], but also [5]). Hence, the subjective decision-making process can be examined by means of fuzzy binary relations, exploring preference relations as gradual predicates [13], where *preference* is represented as a linguistic variable in the sense of [43].

From this standing point (see again [10,26,40]), the characterization of the fuzzy weak preference relation is given by

$$R(a,b) = \{\langle a,b,\mu_R(a,b)\rangle | a,b \in A\}, \tag{5}$$

where $\mu_R(a,b) \in L$ is the membership intensity for every $(a,b) \in A \times A$, according to the verification of the property "being at least *as desired* as", where L is a completely ordered linguistic scale. In this way, the individual is able to develop a linguistic preference evaluation, by means of a linguistic term or word, for expressing the degree or intensity of desire over any pair of alternatives.

In consequence, following a linguistic symbolic computational model based on ordinal scales and max–min operators [42], the preference predicate is valued over an scale L composed by distinctive linguistic terms (see, e.g., [16,25]). Here, we choose the granularity of the linguistic term set L assuming that its elements, i.e., linguistic terms, are *symmetrically* distributed, such that

$$L = \{l_1 = null, l_2 = low, l_3 = medium, l_4 = high, l_5 = absolute\}, \tag{6}$$