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A linguistic multi-criteria decision making approach based on logical reasoning

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ABSTRACT

In real decision making problems, it is always more natural for decision makers to use linguistic terms to express their preferences/opinions in a qualitative way among alternatives than to provide quantitative values. Additionally, many of these decision making problems are under uncertain environments with vague and imprecise information involved. Following the idea of Computing with Words (CWW) methodology, we propose in this paper a linguistic valued qualitative aggregation and reasoning framework for multi-criteria decision making problems, where a linguistic valued algebraic structure is constructed for modelling the linguistic information involved in multi-criteria decision making problems, and a linguistic valued logic based approximate reasoning method is developed to infer the final decision making result. This method takes the advantage of handling the linguistic information, no matter totally ordered or partially ordered, directly without numerical approximation, and having a non-classical logic as its formal foundation for decision making process.

1. Introduction

Decision making, which, in many cases, can be seen as the process for choosing the most appropriate one among a set of alternatives under provided criteria or preferences, is the crucial step in many real applications such as organization management, financial planning, risk assessment, products evaluation and recommendation. Qualitative information is frequently used in the area of decision-making, such as judgments/opinions from experts, which are always expressed by linguistic terms in natural language. Linguistic terms, not like numerical ones whose value are crisp numbers, are always vague and imprecise [13,14,25,31,34,37]. For example, when we are evaluating the quality of a computer, which is qualitative in nature, the evaluations are usually expressed as linguistic terms, such as "*satisfied*," "*acceptable*," or "*good*". Computing with Words (CWW) [39,46] methodology has been the most popular one to model and manipulate linguistic information to solve decision making problems under qualitative and uncertain environment.

The main inspiration of CWW comes from the human ability to deal with different uncertain, incomplete and imprecise perceptions naturally without needing an explicit use of any measurements or computations. Therefore, CWW aims at manipulating perceptions, or words and propositions from natural language, which contrasts to the usual sense of

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computing, i.e., manipulating numbers, and providing a methodology which can enhance computer's ability to deal with imprecision, uncertainty and partial truth which is pervasive in human daily recognition and decision processes [14,31].

The conventional CWW methodology is based on fuzzy set theory [2,35,38,39,46], which generally converts the linguistic information being processed into a fuzzy set (FS) and modelled by membership function or fuzzy number, and a CWW engine, *perceptual computer*, then works to map these FSs into some other FS, after which the generated FS will be transformed back into a word or proposition using linguistic approximation [33]. The process of converting words to FSs and then FSs to words again, which is mainly based on the extension principle, is usually time consuming, computationally complex, and involving loss of information [17]. Furthermore, it is not always an easy task to find the feasible FSs for representing the corresponding words. Taking into account the fact that human beings manipulate the perceptions directly without needing to convert them into numerical values, and the process is always accompanied with reasoning, it would be more natural and reasonable to represent and reason about linguistic information in its original form, i.e., through the symbolic way [42].

Symbolic approaches [1,12,30] use linguistic symbols (usually with indexes) to represent linguistic information directly without the numerical approximation required by fuzzy set based methods, and aggregate or compute on the indexes of these symbols to obtain the final result. One of the representative symbolic linguistic information processing approaches, fuzzy ordinal linguistic approach [12,18,31], uses an ordered structure, linguistic labels with indexes, to represent the set of linguistic terms, with the assumption that the terms under discussion is totally ordered [32]. The 2-tuple linguistic representation (or computational) model [16,17,30], a continuous linguistic representation and computation model, is one of most popular extended fuzzy ordinal linguistic approaches. In this model, the linguistic 2-tuple, a pair of values, (L_i, α_i), is used to represent the linguistic information, where $L_i \in S$ is a linguistic label and the number $\alpha_i \in [-0.5, 0.5)$ is called the symbolic translation, which supports "difference of information" between the result obtained after aggregation and the closest one in the set of linguistic terms.

Although fuzzy ordinal linguistic approaches take the advantages of without loss of information and computational simplicity by avoiding use of membership function [17,30], it requires that the linguistic information is totally ordered and can be manipulated by indexes. This limits the application of fuzzy ordinal linguistic approach to more general situations where partially ordered information often involved.

Partially ordered information is ubiquitous in our daily decision making problems because of the uncertain and dynamic environment [29]. For example, one alternative is better in one aspect but may be worse in another, and we always find it difficult to make a decision when multiple criteria are involved where conflicting opinions always exist. Partial orders are more flexible than total orders to represent incomplete, uncertain and imprecise knowledge. Moreover, they avoid comparing unrelated pieces of information which is required by total order based approaches.

Lattice, a special kind of partially ordered structure, has been shown to be an appropriate and efficient structure for representing ordinal qualitative information in the real world due to its additional operations and better properties [6,23]. Mainly from the algebraic point of view, Ho et al. [19–22] proposed an algebraic structure for modelling linguistic terms, Hedge algebra. Hedge algebra consists of two parts: linguistic hedges and prime terms (also called generators), where linguistic hedges are actually some kind of linguistic modifiers, e.g., "*more or less*," "*quite*," "*highly*," on the prime terms such as "*true* and *false*," "*high* and *low*". These linguistic hedges take the role in strengthening or weakening the meaning of prime terms. By applying the set of hedges to the prime terms, Hedge algebra is then constructed which is essentially a partially ordered structure according to the natural meanings of the represented linguistic terms, and generally a lattice as Fig. 1.1 shows.

Although Hedge algebra is able to reflect the semantic ordering relation, partial order in many cases, among the considered linguistic terms [22], and has a close relation with logic systems and approximate reasoning methods due to the fact that it is based on lattice structure, there is no logic system and the corresponding approximate reasoning methods has been built based on Hedge algebras [9].

In our view, the process of decision making can be essentially interpreted as a reasoning process from provided information or domain knowledge to some conclusion, and decision making under qualitative and uncertain environments is essentially an approximate reasoning process. From the viewpoint of symbolism, the confidence and rationality of certain reasoning is rooted in the solid classical logic foundation. Accordingly, the confidence and rationality of approximate reasoning then relies on non-classical logics [7,36], which are extensions of the classical logic. Therefore, it is important and necessary to study the rational logic-based approximate reasoning approach for decision making problems under uncertain environment [9,42].

Logic generally can be used for modelling decision making problems in two different ways: syntactic and semantic [9]. From the syntactic point of view, logic uses formulas and propositions to represent judgments from decision makers. For example, the judgments of a set of decision makers among a set of alternatives can be represented by the propositions of a logic system, e.g., p_1, p_2. Such as, p_1 means that alternative 1 performs well in some specified property. The composite propositions, which are composed by the primitive propositions p_1, p_2, etc. with logical connectives $'$ (not), $\bigwedge$ (and), $\bigvee$ (or), $\rightarrow$ (if-then) and $\leftrightarrow$ (if and only if), can be used for modelling more complex judgments. Then different logical reasoning methods, such as fuzzy Modus Ponens (MP) rule and fuzzy Compositional Rule of Inference (CRI) [46], are applied to reach the collective evaluation. From the semantic side, the truth-value field of logic system, such as $\{0,1\}$ for classical logic, or $[0,1]$ for fuzzy logic, is used for modelling the set of evaluations on the alternatives. Take Fig. 1.1 as a truth-value field example, the truth-value of p_1 is *ATrue* means that the judgment of the decision maker on the first alternative is *approximately* true. This kind of truth-value will change accordingly along with the syntactic inference process.