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A new linguistic computational model based on discrete fuzzy numbers for computing with words

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ABSTRACT

In recent years, several different linguistic computational models for dealing with linguistic information in processes of computing with words have been proposed. However, until now all of them rely on the special semantics of the linguistic terms, usually fuzzy numbers in the unit interval, and the linguistic aggregation operators are based on aggregation operators in $[0,1]$. In this paper, a linguistic computational model based on discrete fuzzy numbers whose support is a subset of consecutive natural numbers is presented ensuring the accuracy and consistency of the model. In this framework, no underlying membership functions are needed and several aggregation operators defined on the set of all discrete fuzzy numbers are presented. These aggregation operators are constructed from aggregation operators defined on a finite chain in accordance with the granularity of the linguistic term set. Finally, an example of a multi-expert decision-making problem in a hierarchical multi-granular linguistic context is given to illustrate the applicability of the proposed method and its advantages.

1. Introduction

Uncertainty is a common factor in a wide range of real-world decision-making problems due to the tough task of handling it properly. This uncertainty often comes from the vagueness of meanings that are used by experts in problems where qualitative information is used. When data are qualitative, the fuzzy linguistic approach is a good tool to model them since these qualitative terms are represented via linguistic variables instead of numerical values (see [33–35]). These linguistic variables belong to a linguistic term set having more or less terms depending on the uncertainty degree, or the granularity of uncertainty, provided by the source of information qualifying an alternative. However, the use of the fuzzy linguistic approach have presented some limitations, specially regarding information modelling and computational processes, the so-called processes of computing with words (CW) (see [9,18,20]). To overcome these limitations, several different linguistic models have been presented: the *symbolic linguistic model* based on ordinal scales by assuming an ordered structure defined among the labels [31,14]; *the linguistic 2-tuple model* [16], which introduces the symbolic translation to the linguistic representation; *the linguistic model based on type-2 fuzzy sets representation* [27], which represents the semantics of the linguistic terms using type-2 membership functions and *the proportional 2-tuple model* [29], which extends the 2-tuple model by using two linguistic terms with their proportion to model the information, among many others.

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However, from an accurate analysis of these models, some common properties can emerge. First of all, there still exists some kind of limitation on the modelling of the linguistic information. The experts must express their evaluations on the alternatives in a single term that should be encompassed into a linguistic term set of a linguistic variable. This fact comes from that qualitative information is often interpreted to take values in a totally ordered finite scale like this:

$$\mathcal{L} = \{Extremely\ Bad, Very\ Bad, Bad, Fair, Good, Very\ Good, Extremely\ Good\}.$$

These linguistic terms will be denoted by *EB*, *VB*, *B*, *F*, *G*, *VG* and *EG*, respectively. In these cases, the representative finite chain $L_n = \{0, 1, \ldots, n\}$ is usually considered to model these linguistic hedges. However, the modelling of linguistic information is limited because the information provided by experts for each variable must be expressed by a simple linguistic term. In most cases, this is a problem because the expert's opinion does not agree with a concrete term. On the contrary, experts' values are usually expressions like *"better than Good"*, *"between Fair and Very Good"* or even more complex expressions. In [25], using the hesitant fuzzy sets introduced by Torra in [26], a new model was proposed to increase the flexibility and richness of linguistic elicitation in hesitant situations under qualitative settings.

The second common property of these models is the use of fuzzy numbers defined in the [0, 1] interval or their extensions to represent the semantics of the linguistic terms. The semantics are useful to capture the vagueness of the linguistic assessments given by the experts. The final remarkable common property is the use of the so-called linguistic aggregation functions (see [10,30]). These operators are used in order to merge the evaluations given by the experts into a representative output. These operators are very useful for modelling those processes in which there are various information sources and the information is linguistic in nature. However, most of these operators rely on an aggregation operator defined on the unit interval. First, the linguistic inputs are transformed into real numbers on the unit interval, the aggregation operator on the unit interval is applied and a transformation is performed in the output to recover a linguistic term. Another approach has been developed by several researchers through an extensive study of aggregation functions on L_n, usually called *discrete aggregation functions* (see [19,21,23]).

Taking into account the previous considerations, we propose a linguistic computational model based on discrete fuzzy numbers whose support is a subset of consecutive natural numbers contained in L_n (i.e, an interval contained in L_n), usually denoted by $\mathcal{A}_1^{L_n}$ (see [28] and [24]). The idea lies on the fact that any discrete fuzzy number $A \in \mathcal{A}_1^{L_n}$ can be considered (identifying the scale $\mathcal{L}$ given previously with $n = 6$) as an assignation of a [0,1]-value to each term in our linguistic scale. As an example, the above mentioned expression *"between Fair and Very Good"* can be performed, for instance, by a discrete fuzzy number $A \in \mathcal{A}_1^{L_6}$, with support given by the subinterval $[F, VG]$ (that corresponds to the interval [3, 5] in L_6). The values of A in its support should be described by experts, allowing in this way a complete flexibility of the qualitative valuation overcoming the limitation on the modelling of the linguistic information. A possible discrete fuzzy number A representing the expression mentioned above is given in Fig. 1 (note that there are pictured only the values of A in its support).

Thus, discrete fuzzy numbers can be interpreted as flexible qualitative information and they have already been successfully used in decision making problems and subjective evaluation in [24]. In addition, in this model, the semantics of the linguistic terms are included into the evaluation of the expert and there is no need of defining any underlying membership functions.

On the other hand, the model is also useful to avoid the limitation of the aggregation functions on L_n or the internal use of aggregation functions into the linguistic aggregation functions. In [3,4,24], the authors deal with the possibility of extending monotonic operations on L_n to operations on the set of discrete fuzzy numbers whose support is a set of consecutive natural numbers $\mathcal{A}_1^{L_n}$. These aggregation functions on $\mathcal{A}_1^{L_n}$ will allow us to manage qualitative information in a more flexible way and preserving all the model in a linguistic context. In [3] t-norms and t-conorms on $\mathcal{A}_1^{L_n}$ are described and studied, as well as it is done for uninorms, nullnorms and general aggregation functions in [24]. In both cases, an example of application in decision making or subjective evaluation is included.

This paper is structured as follows. In Section 2, we make a brief review of discrete fuzzy numbers and the linguistic hierarchical structure. In Section 3, the linguistic model based on discrete fuzzy numbers is proposed for a multiple expert decision making problem defined over a subjective linguistic hierarchy. Finally, in Section 4, we present an example of a multi-

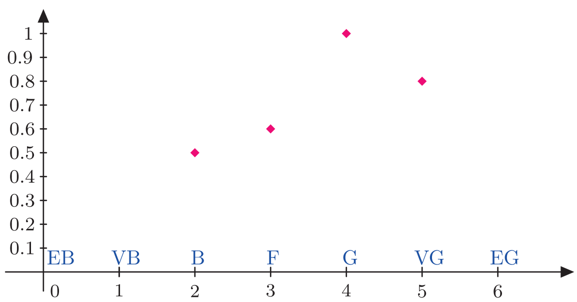

Fig. 1. Graphical representation of a discrete fuzzy number whose support is the interval [2,5]. In addition, note that this fuzzy set can be interpreted as expression "between Bad and Very Good", after identifying the linguistic scale $\mathcal{L}$ with the chain L_6.