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How to measure the amount of knowledge conveyed by Atanassov's intuitionistic fuzzy sets ☆

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ABSTRACT

We address the problem of how to measure the amount of knowledge conveyed by the Atanassov's intuitionistic fuzzy set (A-IFS for short). The problem is relevant from the point of view of many application areas, notably decision making. An amount of knowledge considered is strongly linked to its related amount of information. Our analysis is concerned with an intrinsic relationship between the positive and negative information and a lack of information expressed by the hesitation margin. Illustrative examples are shown.

1. Introduction

In the case of data represented in terms of fuzzy sets, information conveyed is expressed by a membership function. On the other hand, knowledge is basically related to information considered in a particular useful context under consideration. The transformation of information into knowledge is critical from the practical point of view (cf. [12]) and a notable example may here be the omnipresent problem of decision making.

In this paper we are concerned with information conveyed by a piece of data represented by an A-IFS, and then its related knowledge that is placed in a context considered. Information that is conveyed by an A-IFS, may be considered just as some generalization of information conveyed by a fuzzy set, and consists of the two terms present in the definition of the A-IFS, i.e., the membership and non-membership functions ("responsible" for the positive and negative information, respectively). But for practical purposes, which can be viewed as following a general attitude to use as much information as is available, it seems expedient, even necessary, to also take into account a so called hesitation margin (cf. [17,19,26,21,28,29,33,5,6,36–38], etc.).

Entropy is often viewed as a dual measure of the amount of knowledge. In this paper we show that the entropy alone (cf. [21,28]) may not be a satisfactory dual measure of knowledge useful from the point of view of decision making in the A-IFS context. The reason is that an entropy measure answers the question about the fuzziness as such but does not consider any peculiarities of how the fuzziness is distributed. So, the two situations, one with the maximal entropy for a membership function equal to a non-membership function (e.g., both equal to 0.5), and another when we know absolutely nothing (i.e., both equal to 0), are equal from the point of view of the entropy measure (in terms of the A-IFSs). However, from the point of view of decision making the two situations are clearly different.

We should have in mind that a properly constructed entropy measure for the fuzzy sets is from the interval $[0,1]$ so we cannot increase the value above "1" while considering the entropy of A-IFSs. Each properly constructed entropy for the A-IFSs, due to the very sense of entropy, should also be from the interval $[0,1]$. The simple way to overcome this situation is to calculate the entropy of the A-IFSs just using an entropy measure for the fuzzy sets, and to add separately a term relate

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d to the hesitation margin. In other words, we might express the entropy via two numbers. But such a measure would be difficult to use while solving real problems (many conditions would be verified to come to a conclusion).

On the one hand, the need for consistency of the entropy for the fuzzy sets and A-IFSs, and the need to express in a simple way differences occurring from the point of view of decision making when the hesitation margin is greater than 0 for the A-IFSs is the motivation of this paper as we propose here a new measure of knowledge for the A-IFSs which is meant to complement the entropy measure to be able to properly capture additional features which may be relevant while making decisions.

We would like to stress that the problem does not boil down to introducing a new entropy measure or to comparing the existing entropy measures. The reason is that properly defined entropy measures for the A-IFSs cannot reflect in a simple way (from the point of view of calculation) an additional piece of information being a result of hesitation margins in the A-IFS context, and being consistent with the values obtained for the fuzzy sets. The new measure of the amount of knowledge is tested on a simple example taken from the source Quinlan's paper [11] but solved using different tools than therein. This example, being simple by just judging by appearance, is a challenge to many classification and machine learning methods. Here we make use of it to verify if it is possible to obtain the same optimal solution obtained by Quinlan while using the new measure of the amount of knowledge. Data to another example we consider comes from the benchmark data known as "Sonar" [40].

2. Brief introduction to the A-IFSs

The concept of a fuzzy set in X [39], given by

$$A' = \{\langle x, \mu_{A'}(x)\rangle | x \in X\} \tag{1}$$

where $\mu_{A'}(x) \in [0, 1]$ is the membership function of the fuzzy set A', can be generalized.

One of well known generalizations is that of an A-IFS [1–3] A which is given by

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\} \tag{2}$$

where $\mu_A: X \to [0,1]$ and $\nu_A: X \to [0,1]$ such that

$$0 \leqslant \mu_A(x) + \nu_A(x) \leqslant 1 \tag{3}$$

and $\mu_A(x)$, $\nu_A(x) \in [0,1]$ denote a degree of membership and a degree of non-membership of $x \in X$, respectively.

An additional concept related to an A-IFS in X, that is not only an obvious result of (2) and (3) but which is also relevant for applications, is called a *hesitation margin* of $x \in A$ given by (cf. [2])

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x) \tag{4}$$

which expresses (a degree of) lack of information of whether x belongs to A or not. It is obvious that $0 \leqslant \pi_A(x) \leqslant 1$, for each $x \in X$.

The hesitation margin (4) turns out to be important while considering the distances [17,19,26,33], entropy [21,25,28], similarity [29] for the A-IFSs, ranking [30,31], etc. i.e., the measures that play a crucial role in virtually all information processing tasks. Hesitation margins turn out to be relevant for applications – in image processing (cf. [5,6]) and classification of imbalanced and overlapping classes (cf. [36–38]), group decision making, negotiations, voting and other situations [16,18,20,22–24,27]).

As we will use three term representation of A-IFSs, each element x will be described via a triple: (μ, ν, π), i.e., by the membership μ, non-membership ν, and hesitation margin π.

The concept of a complement of an A-IFS, A, is clearly crucial. It is denoted by A^C, and defined usually, also here, as (cf. [2]):

$$A^C = \{\langle x, \nu_A(x), \mu_A(x), \pi_A(x)\rangle | x \in X\} \tag{5}$$

2.1. Two geometrical representations of the A-IFSs

Having in mind that for each element x belonging to an A-IFS A, the values of membership, non-membership and the intuitionistic fuzzy index sum up to one, i.e.,

$$\mu_A(x) + \nu_A(x) + \pi_A(x) = 1 \tag{6}$$

and that each one of the membership, non-membership, and the intuitionistic fuzzy index are from [0,1], we can imagine a unit cube (Fig. 1) inside which there is an *MNH* triangle where the above equation is fulfilled. In other words, the *MNH* triangle represents a surface where coordinates of any element belonging to an A-IFS can be represented. Each point belonging to the *MNH* triangle is described via three coordinates: (μ, ν, π). Points M and N represent crisp elements. Point $M(1,0,0)$ represents elements fully belonging to an A-IFS as $\mu = 1$. Point $N(0,1,0)$ represents elements fully not belonging to an A-IFS as $\nu = 1$. Point $H(0,0,1)$ represents elements about which we are not able to say if they belong or not belong to an A-IFS (intuitionistic fuzzy index $\pi = 1$). Such an interpretation is intuitively appealing and provides means for the representation of