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A biparametric similarity measure on intuitionistic fuzzy sets with applications to pattern recognition

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ABSTRACT

Unlike an ordinary fuzzy set, the concept of intuitionistic fuzzy set (IFS), characterized both by a membership degree and by a non-membership degree, is a more flexible way to capture the uncertainty. One of the important topics in IFS is the measure of the similarity between IFSs for which several studies have been proposed in the literature. Some of those, however, cannot satisfy the axioms of similarity, and provide counter-intuitive cases. In this paper, a new general type of similarity measure for IFS with two parameters is proposed along with its proofs. A comparison between the existing similarity measures and the proposed similarity measure is also performed in terms of counter-intuitive cases. The findings indicate that the proposed similarity measure does not provide any counter-intuitive cases.

1. Introduction

The theory of fuzzy set, proposed by Zadeh [1], has received a great deal of attention due to its capability of handling uncertainty. Therefore, over the last decades, several higher order fuzzy sets have been introduced in the literature. Intuitionistic fuzzy set (IFS), proposed by Atanassov [2], is one of the higher order fuzzy sets which is capable of dealing with vagueness. An IFS is characterized by three parameters, namely a membership degree, a non-membership degree, and a hesitation margin, while a fuzzy set is characterized by only a membership degree. IFS is therefore a more effective way to deal with vagueness than fuzzy set. Although Gau and Buehrer [3] later presented vague set, Bustince and Burillo [4] pointed out that the notion of vague sets was the same as that of IFS.

The degree of similarity measure has received a great deal of attention in the last decades since it is an important tool for decision making, pattern recognition, medical diagnosis, and the applications of data mining [5]. For that reason, some studies on the measure of similarity between IFSs have been reported in the literature. A few of them is the extension of the well-known distance measures. The first study was carried out by Szmidt and Kacprzyk [6] extending the well-known distances measures, such as the Hamming Distance, the Euclidian Distance, to IFS environment and comparing them with the approaches used for ordinary fuzzy sets. However, Wang and Xin [7] implied that the distance measure of Szmidt and Kacprzyk [6] were not effective in some cases. Therefore, several new distance measures were proposed and applied to pattern recognition. Grzegorzewski [8] also extended the Hamming distance, the Euclidean distance, and their normalized counterparts to IFS environment. Later, Chen [9] pointed out that some errors existed in Grzegorzewski [8] by showing some counter examples. Hung and Yang [10] extended the Hausdorff distance to IFSs and proposed three similarity measures. On the other hand, instead of extending the well-known measures, some studies defined new similarity measures for IFSs. Li and Cheng [11] suggested a new similarity measure for IFSs based on the membership degree and the non-membership degree. Mitchell

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[12] showed that the similarity measure of Li and Cheng [11] had some counter-intuitive cases and modified that similarity measure based on statistical point of view. Moreover, Liang and Shi [13] presented some examples to show that the similarity measure of Li and Cheng [11] was not reasonable for some conditions, and therefore proposed several new similarity measures for IFSs. Li et al. [14] analyzed, compared and summarized the existing similarity measures between IFSs/vague sets by their counter-intuitive examples in pattern recognition. Ye [15] conducted a similar comparative study of the existing similarity measures between IFSs and proposed a cosine similarity measure and a weighted cosine similarity measure. Hwang et al. [16] proposed a similarity measure for IFSs in which Sugeno integral was used for aggregation. The proposed similarity measure was applied to clustering problem. Xu [17] introduced a series of similarity measures for IFSs and applied them to multiple attribute decision making problem based on intuitionistic fuzzy information. Xu and Chen [18] introduced a series of distance and similarity measures, which are various combinations and generalizations of the weighted Hamming distance, the weighted Euclidean distance and the weighted Hausdorff distance. Xu and Yager [19] developed a similarity measure between IFSs and applied the developed similarity measure for consensus analysis in group decision making based on intuitionistic fuzzy preference relations. Xia and Xu [5] proposed a series of distance measures based on the intuitionistic fuzzy point operators. In addition to these studies, some works have been interested in relationships between distance measure, similarity measure and entropy of IFSs. Zeng and Guo [20] investigated the relationship among the normalized distance, the similarity measure, the inclusion measure, and the entropy of interval-valued fuzzy sets. It was also showed that the similarity measure, the inclusion measure, and the entropy of interval-valued fuzzy sets could be induced by the normalized distance of interval-valued fuzzy sets based on their axiomatic definitions. Wei et al. [21] introduced a entropy measure generalizing the existing entropy measures for IFS and IFSs. It was also introduced an approach to construct similarity measures using entropy measures for IFS and IFSs.

In this paper, we introduce a new distance measure between IFSs and give its relation with the similarity measure for IFSs. The proposed generalized distance measure on intuitionistic fuzzy sets be presented in Eq. (8) depends on two parameters where p is the L_p norm and t identifies the level of uncertainty. We compare the existing similarity measures with the proposed similarity measure for IFSs. In order to do so, the rest of this paper is organized as follows. Section 2 presents the definitions related to the IFSs, and lists the properties that a distance measure for IFSs and a similarity measure for IFSs should possess. The new distance measure and corresponding novel type of similarity measure are expressed in Section 3. The interpretation of new distance measure and the explanation of its parameter are briefly introduced in Section 4. A comparative analysis between the proposed similarity measure and the existing similarity measures is presented in Section 5. The applications of the proposed similarity measure to pattern recognition are presented in Section 6. The conclusion of the paper is given in Section 7.

2. Preliminaries

In this section, we briefly introduce the basic concepts related to IFS, and then list the properties that a distance measure for IFSs and a similarity measure for IFSs should possess.

Definition 1 [1]. A fuzzy set A in the universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$ is defined as follows:

$$A = \{\langle x, \mu_A(x)\rangle | x \in X\} \tag{1}$$

where $\mu_A(x)$: $X \to [0,1]$ is the membership degree.

Definition 2 [2]. An IFS A in a finite set X can be written as:

$$A = \{\langle x, \mu_A(x), v_A(x)\rangle | x \in X\} \tag{2}$$

where $\mu_A(x)$: $X \to [0,1]$ and $v_A(x)$: $X \to [0,1]$ are membership degree and non-membership degree, respectively, such that:

$$0 \leqslant \mu_A(x) + v_A(x) \leqslant 1 \tag{3}$$

The third parameter of the IFS is:

$$\pi_A(x) = 1 - \mu_A(x) - v_A(x) \tag{4}$$

which is known as the intuitionistic fuzzy index or the hesitation degree of whether x belongs to A or not. It is obviously seen that for every:

$$0 \leqslant \pi_A(x) \leqslant 1 \tag{5}$$

If $\pi_A(x)$ is small, then knowledge about x is more certain; if $\pi_A(x)$ is great, then knowledge about x is more uncertain. Obviously, when $v_A(x) = 1 - \mu_A(x)$ for all elements of the universe, the ordinary fuzzy set is recovered [22].

Definition 3. Let $\tilde{a} = (\mu_a, v_a)$ be an intuitionistic fuzzy number (IFN), then the score function of $\tilde{a}$ where $s(\tilde{a}) \in [-1, 1]$ is defined as follows [23]: