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Aggregation functions for typical hesitant fuzzy elements and the action of automorphisms

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ABSTRACT

This work studies the aggregation operators on the set of all possible membership degrees of typical hesitant fuzzy sets, which we refer to as $\mathbb{H}$, as well as the action of $\mathbb{H}$-automorphisms which are defined over the set of all finite non-empty subsets of the unitary interval. In order to do so, the partial order $\leqslant_{\mathbb{H}}$, based on α-normalization, is introduced, leading to a comparison based on selecting the greatest membership degrees of the related fuzzy sets. Additionally, the idea of interval representation is extended to the context of typical hesitant aggregation functions named as the $\mathbb{H}$-representation. As main contribution, we consider the class of finite hesitant triangular norms, studying their properties and analyzing the $\mathbb{H}$-conjugate functions over such operators.

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1. Introduction

The fuzzy set theory was introduced as a mathematical framework to deal with the incompleteness of information of real systems and the necessity of combining granularity and flexibility in the representation of such information in practical reasoning tasks. Since first introduced by Zadeh [44], many extensions of the fuzzy set theory have been conceived.

Type-2 Fuzzy Sets (T2FSs) are an important generalization of classical fuzzy sets able to model vague concepts via more flexible (non-precise) membership functions. Some of the main results on this generalization are summarized in [18,19]. Their flexibility in the representation of the ambiguity comes coupled to severe problems in their practical applications. Interval-valued fuzzy sets (IVFSs) were conceived [30,45] as a particular class of T2FSs which captures the imprecision of the membership degree as an interval, reflecting the measure of vagueness and uncertainty in the width of such intervals. Further significant results are also Atanassov's intuitionistic fuzzy sets [1] (AIFSs), taking into account concepts of intuitionistic logic by considering the hesitation related to the dual construction of (non-)membership degrees. See more details in [2,4]. An integrated approach, named as the interval-valued intuitionistic fuzzy set theory [3] is born from the combination of the concept of IVFS and AIFS by relaxing the complementary operation and modelling the membership degrees by means

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of intervals or Atanassov's intuitionistic pairs. Another direction of research, but with analogous results, considers vague sets (see, e.g., [8,22]).

On the hall of approaches related to many-valued fuzzy sets, this work focuses on the study of Hesitant Fuzzy Logic (HFL) whose foundations come from the theory of Hesitant Fuzzy Sets (HFSs). This theory was recently introduced in [33,34] as an appropriate tool to deal with multi-criteria decision making. Thus, it would be possible to take a set of values grouped together based on certain criteria, in order to define the membership degree of an element in a HFS.

1.1. Related works on aggregating information based on HFSs

Very frequently, distinct approaches to deal with multi-criteria decision making use aggregation operators to group the information prior to the reasoning phase. These operators have an important role in fuzzy reasoning, as presented in [10,11,15,17,18,46,47]. Very relevant examples of aggregation operators often used for decision making are the Weighted Averaging (WA) and Ordered Weighted Averaging (OWA) operators, which are based on weighting vectors, and their corresponding continuous extensions C-WA and C-OWA [43]. These operators have been subsequently generalized to produce aggregated information based on confidence indexes and weighting vectors, giving rise to Confidence Induced Weighted Aggregation (CIWA) and Confidence Induced Ordered Weighted Aggregation (CIOWA) operators [38] (a review of such operators is included in Section 2).

Moreover, different versions of these operators have been proposed thereafter to aggregate information provided as non-classical fuzzy sets. By using these operators, decision making algorithms are able to deal with different representations of uncertainty. Regarding HFSs, Xia et al. propose a series of such aggregation operators so that they can be used in situations in which there exist difficulties in expressing the membership degree of an element as a scalar value. Further studies [40,42] have produced a wide family of aggregation operators and concepts of entropy and cross-entropy for hesitant fuzzy information are discussed including their desirable properties. In [13], some correlation coefficient formulas for HFSs are derived and applied to clustering analysis under hesitant fuzzy environments. In [48], extensions for the hesitant context of the Bonferroni mean and some of their variants were proposed and applied in multi-criteria decision making.

Research on HFSs has also explored the application of aggregation operators with purposes other than WA operators. In [28], the concept of hesitant fuzzy linguistic term set was introduced to manage hesitation in qualitative contexts and applied in group making decision in [29]. Interesting results related to distances and other similar measures for HFSs are presented in [41]. Additionally, in [37], aggregation operators for HFSs were introduced along with the relationship between HFSs and AIFSs. In [20], a mutual transformation of the entropy into the similarity measure for HFSs was proposed and a partial order on HFSs based on the normalization of hesitant fuzzy elements was also proposed in [41].

More recently, aggregation operators defined over HFSs and their application to multiple attribute decision making are studied in [26,35,36,39]. Furthermore, interval-valued hesitant preference relations were introduced in [12] describing uncertain evaluation information in group decision making processes.

1.2. Relevance of new aggregation functions for HFSs by integration of formal concepts from Fuzzy Logic and Lattice Theory

It is well known that the information provided by inference systems modelled by the Fuzzy Logic and founded on the Fuzzy Set Theory can be formally discussed and compared in terms of the partial ordered sets defined in accordance with the Lattice Theory. In lattice-valued fuzzy set theory, aggregation functions are increasing operators with respect to the order of the lattice [24,25].

Despite the diversity in the above literature describing on aggregation information related to the set $\mathbb{H}$ of hesitant fuzzy sets (HFSs), except the recent results presented in [20] dealing with a partial order relation on HFSs, the major contributions have considered only relations based on distance or score functions to order the hesitant fuzzy elements. As one can easily notice, they are not partial orders since there exist at least two hesitant fuzzy elements associated to the same score image. Moreover, these approaches cannot provide an explicit definition of a hesitant aggregation function (HAF) on $\mathbb{H}$. Consequently, they can be applied to many distinct scenarios but they are only able to specify aggregation operators on $\mathbb{H}$ which extend some well known aggregation function on $[0,1]$.

Differently from such literature which define specific HAFs even without a formal concept of an HAF, in this paper the definition of HAFs is consistent with the definition of aggregation functions valued in the complete bounded lattice $(\mathbb{H}, \leqslant_{\mathbb{H})})$, whenever the partial order $\leqslant_{\mathbb{H}}$ is fixed according to [24,25]. As the main benefit, the minimum criteria for a multidimensional HFA $F : \mathbb{H}^n \rightarrow \mathbb{H}$ can be formalized.

Founded on the Lattice Theory, distinct ways to obtain partial ordered HFSs on $\mathbb{H}$ are presented, which are based on two general normalization principles. By fixing one of this normalization and defining a binary relation $\leqslant_{\mathbb{H}}$ as the more intuitive partial order on $\mathbb{H}$, we are able to compare (by reporting to the usual order on $[0,1]$) one by one all the corresponding most relevant elements of a compatible pair of normalized hesitant fuzzy sets. So, when an HAF is formally defined according to the bounded lattice-valued fuzzy aggregation functions, as considered in [24], we obtain the definition of a hesitant t-norm in $(\mathbb{H}, \leqslant_{\mathbb{H}})$ consistently with the definition of a valued t-norm in $([0,1], \leqslant)$. Inspired by the OWA-like operators, many operators have been defined for the context of HFS theory. In addition, four classes of OWAs are reported in order to show that OWA operators can also be defined on the lattice $(\mathbb{H}, \leqslant_{\mathbb{H}})$.

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