Information Sciences 327 (2016) 21–39

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Finite dimensional guaranteed cost sampled-data fuzzy control for a class of nonlinear distributed parameter systems

Zi-Peng Wang, Huai-Ning Wu*

Science and Technology on Aircraft Control Laboratory, School of Automation Science and Electrical Engineering, Beihang University, Beijing 100191, China

ARTICLE INFO

Article history:
Received 25 December 2014
Revised 12 June 2015
Accepted 9 August 2015
Available online 13 August 2015

Keywords:
Guaranteed cost control
Sampled-data control
Fuzzy control
Distributed parameter system
Bilinear matrix inequality (BMI)

ABSTRACT

In this paper, a finite dimensional guaranteed cost sampled-data fuzzy control (GCSDFC) problem is addressed for a class of nonlinear parabolic partial differential equation (PDE) systems. Initially, applying the Galerkin's method to the PDE system, a nonlinear ordinary differential equation (ODE) system is derived. The resulting nonlinear ODE system can accurately describe the dominant dynamics of the PDE system, which is subsequently expressed by the Takagi–Sugeno (T–S) fuzzy model. Then, a guaranteed cost sampled-data fuzzy controller is developed to stabilize exponentially the closed-loop slow fuzzy system while providing an upper bound for the quadratic cost function. A novel time-dependent functional is constructed to derive the condition for the existence of the proposed controller which is presented by bilinear matrix inequalities (BMIs). Moreover, a suboptimal GCSDFC problem to minimize the cost bound can be formulated as a BMI optimization problem. A local optimization algorithm that views the BMI as a double linear matrix inequality (LMI) is given to solve this BMI optimization problem, in which a Latin hypercube sampling (LHS) method is proposed to find an initially feasible solution for starting the algorithm. Furthermore, it is shown that the proposed controller can ensure the exponential stability of the closed-loop PDE system. Finally, simulation results on the Fisher equation and the temperature profile of a catalytic rod show that the proposed design strategy is effective.

1. Introduction

Since it can take the advantages of both linear system theory and fuzzy logic theory, the Takagi–Sugeno (T–S) fuzzy control approach has been widely used for the controller synthesis of various nonlinear systems over the past few decades (see, e.g., [2–4,6,16,22,26,33,35,36]). However, the existing studies are mainly developed for nonlinear ordinary differential equation (ODE) systems. In practice, the behavior of most industrial processes must depend on time and spatial position, such as fluid flow, heat conduction, elastic wave, and chemical reactor processes. These spatially distributed processes can be described by nonlinear partial differential equations (PDEs) with homogeneous or mixed boundary conditions. Since PDE systems are infinite dimensional systems, the existing T–S fuzzy control approaches for nonlinear ODE systems may not be directly applicable to the control design of nonlinear PDE systems. Until now, the modeling and control of nonlinear PDE systems remains an open and challenging issue.

* Corresponding author. Tel.: +86 1082317301; fax: +86 1082317332.
E-mail addresses: wzp182475@163.com (Z.-P. Wang), whn@buaa.edu.cn, huainingwu@163.com (H.-N. Wu).

http://dx.doi.org/10.1016/j.ins.2015.08.009

As is well known, according to the properties of the spatial differential operator (SDO), PDE systems could be classified as elliptical equation, hyperbolic equation, and parabolic equation, etc. The control of parabolic PDE systems has been studied extensively in the past two decades (e.g., [1,9–11,13–15,25,32]). The results of [1,9–11,13–15,25,32] have involved linear parabolic PDE systems [13,14,32], quasi-linear parabolic PDE systems [1,10,15], and nonlinear parabolic PDE systems [9,11,25]. For parabolic PDE systems, the eigenspectrum of the SDO can be separated into an infinite dimensional stable fast complement and a finite dimensional slow one. Based on the application of the spatial discretization methods (predominantly Galerkin's technique), the proposed finite dimensional ODE systems can exactly describe the dominant dynamic behavior of parabolic PDE systems, which is then used for the basis of the finite dimensional controller design. Recently, those mentioned above T–S fuzzy control technique has been successfully applied to the finite dimensional fuzzy controller synthesis of nonlinear parabolic PDE systems in [5,7,37,39,40,44]. A finite dimensional constrained fuzzy control with guaranteed cost and a multiobjective optimization based fuzzy control are introduced for a class of nonlinear distributed process systems in [37,40], respectively. In [7,39], the fuzzy observer-based control problems for nonlinear parabolic PDE systems are studied. Nonlinear parabolic PDE systems that involve stochastic process and time delay via fuzzy control are proposed in [5,44], respectively. However, it is worth pointing out that the existing studies mainly concentrate on controlling continuous-time physical systems via continuous-time controllers.

On the other hand, the digital controllers have many merits in speed, small size, accuracy, and low price, which are widely used to control various continuous-time systems in the modern control process. When a continuous-time plant is controlled by a digital control algorithm, the closed-loop system is known as a sampled-data system. Three main approaches have been developed to control sampled-data systems which are described by ODEs: the discrete-time system approach [18,31,34], the time-delay system approach [17,29], and the impulsive system approach [8,30]. Recently, these approaches have been extended to the T–S fuzzy ODE systems with sampled-data control in the past few years ([20,23,24,41,43,45], and references therein). For example, the time-delay system approach is introduced to sampled-data fuzzy control for nonlinear ODE systems in [20,24,43]. Then, an improved input delay approach to stabilization of fuzzy ODE systems under variable sampling is proposed based on a novel Lyapunov–Krasovskii functional (LKF) in [45]. These results of [20,24,43,45] are further improved in [41] and successfully applied to chaotic systems. More recently, the sampled-data fuzzy control design for T–S model-based fuzzy ODE systems using the discrete-time system approach is put forward in [23]. It is noted that most of the existing results on sampled-data fuzzy control focus on nonlinear ODE systems. As mentioned earlier, the behavior of most industrial processes is described by nonlinear PDE systems that play an important role in engineering applications. However, the existing studies for PDE systems mainly design a continuous-time controller to control the continuous-time plant. To fully enjoy the benefits of the digital technology in control engineering, it is of importance to develop sampled-data control methods for PDE systems. Unfortunately, there are very few results focusing on sampled-data control design for nonlinear parabolic PDE systems, which motivates the present study.

In this study, a finite dimensional sampled-data fuzzy control approach will be developed for a class of nonlinear parabolic PDE systems. Galerkin's technique is initially applied to the PDE system to obtain a nonlinear slow system of finite dimensional ODEs. Subsequently, the T–S fuzzy model is used to describe exactly the nonlinear slow system. Then, a guaranteed cost sampled-data fuzzy controller design is proposed such that the closed-loop slow system is exponentially stable and an upper bound of the quadratic cost function is provided. The condition for the existence of the proposed controller is derived based on a novel time-dependent functional and presented by bilinear matrix inequalities (BMIs). Moreover, a suboptimal GCSDFC problem to minimize the cost bound can be formulated as a BMI optimization problem. Based on the existing LMI optimization techniques [12,19], a local optimization algorithm that views the BMI as a double LMI is also given to solve this BMI optimization problem, where a Latin hypercube sampling (LHS) method is proposed to find an initially feasible solution for starting the algorithm. Furthermore, it is proven that the proposed controller can ensure the exponential stability of the closed-loop PDE system. Finally, the proposed design strategy is successfully employed to the control of the Fisher equation and the temperature profile of a catalytic rod.

The main contribution and novelty of this paper are summarized as follows: (i) A novel time-dependent functional is constructed to finite dimensional guaranteed cost sampled-data fuzzy control design for a class of nonlinear parabolic PDE systems; (ii) Based on the time-dependent functional technique, LHS method, and the LMI technique, a suboptimal GCSDFC problem is formulated as a BMI optimization problem, which can be solved by a local optimization algorithm.

Notations: $\mathbb{N}$ is the set of nonnegative integers. $\mathbb{R}_+$, $\mathbb{R}$ denote the set of nonnegative real and real numbers, respectively. $\mathbb{R}^n$, $\mathbb{R}^{n\times m}$ denote the n-dimensional Euclidean space and the set of all real $n \times m$ matrices, respectively. l^2 denotes the subset of $\mathbb{R}^\infty$ consisting of all square summable infinite sequences of real numbers, such that $l^2 = \{x = [x_1 \cdots x_\infty]^{\mathrm{T}} \in \mathbb{R}^\infty \,|\, \|x\|_{l^2} < \infty\}$ where $\|x\|_{l^2} \triangleq \sqrt{\sum_i^\infty x_i^2}$. For given constants $\alpha, \beta \in \mathbb{R}$, $L^2([\alpha, \beta]; \mathbb{R}^n)$ is a space of piecewise continuous, square-integrable vector functions defined on the interval $[\alpha, \beta]$. $L^2([\alpha, \beta]; \mathbb{R}^n) \triangleq \{\zeta : [\alpha, \beta] \to \mathbb{R}^n$ and $\|\zeta\|_{2,[\alpha\beta]} < \infty\}$ with the norm $\|\zeta\|_{2,[\alpha\beta]} \triangleq \sqrt{\int_\alpha^\beta \zeta^{\mathrm{T}}(z)\zeta(z)dz}$. $\|\cdot\|$ denotes the Euclidean norm for vector or the spectral norm of matrices. For a symmetric matrix M, $M \geq 0\,(>0, \leq 0, <0)$ means that it is positive-semidefinite (positive-definite, negative-semidefinite, negative-definite, respectively). $\underline{\sigma}(\cdot)$, $\bar{\sigma}(\cdot)$ denote the minimum singular value and the maximum singular value of a matrix, respectively. diag$\{\cdot\}$ stands for a diagonal block matrix. 0_n and I_n denote the $n \times n$ zero matrix and $n \times n$ identity matrix, respectively. M^{T} stands for the transpose of the matrix M and a symmetric matrix by $\begin{bmatrix} A & B \\ B^{\mathrm{T}} & C \end{bmatrix} = \begin{bmatrix} A & B \\ * & C \end{bmatrix}$.