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# A Retinex model based on Absorbing Markov Chains

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ABSTRACT

The Retinex algorithm, developed by Land and McCann, provides an abstract model of the mechanism of color sensation in the Human Vision System. At the basis of model lies the fact that the color appearance of a point does not depend only on its color value, but rather on the comparison among itself and other pixels. According to the model, separately for each chromatic channel, an image pixel receives suitably filtered information about the brightness of other image regions, based on which its own brightness is eventually re-scaled. The original formulation (Land and McCann, 1971) uses a *path-based sampling* approach: the information is transported by memoryless random walks, starting from randomly chosen points; along the path the information is filtered – based on the brightness of the travelled regions – by a specific path function, computed through chains of ratios of pixel intensities. Such a function is path-dependent and retains the value of the brightest point found along the path. The overall correction to a pixel depends on the specific realizations of two sampling processes: the *starting-point sampling* process and the *path-sampling* process. As a consequence of the sampling, this algorithm is known to be intrinsically noisy. This draw-back can be overcome by passing from the path-sampling algorithm to the probabilistic representation of the corresponding diffusion process. In this paper we start from the random path simulative model of Retinex, we respell the standard path-based sampling process representation of the Retinex model, as formalized in Provenzi et al. (2005), and we show that – despite the overall path-dependence – the model can be given a representation in terms of Absorbing Markov Chains, by means of the embedding into a suitable state-space. We derive the corresponding analytic model, accounting for the combined effects of path-function, path sampling process and starting-point sampling process. Finally we provide a numerical algorithm for working out its solution. Using such a model, the output brightness of a pixel can be computed based on the solution of a simple sparse linear system. We show that the output of the random walk sampling algorithm and the Markov Chain based algorithm agree to an extent that can be controlled by few model parameters. We have found also that the Markov Chain based algorithm is more efficient than the basic random path sampling in obtaining noise free images. Those analytic probabilistic models and simulative models can be used as complementary tools for studying the Retinex mechanism and for identifying and comparing variants.



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## 1. Introduction

For Newton and many scholars after him, the sensation of color was a direct derivation of the wavelength composition of the visual stimulus. Many years and experiments later, this basic mechanism has been better understood. Color sensation, and consequently color appearance, are not merely based upon the color stimulus at the point, but rather upon the relative spatial arrangement of the stimuli in the observed scene [1]. This is a more complex mechanism with respect to the simple transformation of the pointwise stimulus, and it allows a quite stable visual perception, less sensitive to variations in intensity and color of the illuminant (this feature is referred to as *color constancy*).

To realize such a robust method, the human vision system (HVS) derives the color sensation comparing spatially the visual stimuli acquired from the scene. The first computational model that formalized the spatial mechanism of human vision was *Retinex*. In a series of papers, Land and McCann [17,18], in order to explain their experimental findings, provided a method to compute color sensation in the form of an algorithm, supposedly operated by a retina-and-cortex system (whence the name *Retinex*).

They structured their algorithm around three principles:

- the HVS operates in three independent retinal-cortical systems, processing respectively the low, middle and high frequencies of the visible spectrum (each system is an instance of *Retinex*);
- each *Retinex* system attenuates smooth changes in intensity – due, for example, to gradients of the illuminant – and enhances sharp changes, i.e. the edges;
- each *Retinex* system estimates the appearance of a point by relating its value to a *local white reference*, then rescales the intensity of the point with respect to that local reference.

On the basis of those principles, Land and McCann devised an algorithm, which processes independently the three channels of an image. In each chromatic channel, the lightness value of an image region is determined as a ratio between its *local intensity* and the ones of the other areas of the image. Their abstract algorithm could be used to estimate such ratios within a single-pass: the algorithm was based on the evaluation of chain products of intensity ratios (in short *ratio-chains*) between contiguous areas, along paths scanning the image. In order to compute the lightness of a target region, the algorithm samples a certain number of paths:

1. for each path computes the ratio-chain;
2. if a ratio deviates from one less than a fixed small threshold value, then the ratio is not used in the computation;
3. if the ratio-chain exceeds the unitary value at a point of the path, then the value is forced to a unitary value, this happens when a lighter local reference region is found along the path;
4. the contribution computed in this way for each individual path is eventually averaged over the set of paths.

Prescription (2) is known as the *threshold mechanism*, prescription (3) is known as the *reset mechanism*.

The algorithm, informally presented by Land and McCann [17] had the goal of modeling the way HVS forms the sensation of color, mimicking the mechanism that generates these independent channel lightnesses. Over the time, it has been found very effective in broader applications of image enhancement, giving rise to a wide family of spatial color processing algorithms [39] with diverse properties and goals. Among the application of Retinex we recall the following: estimating reflectance at a point [15,16], HDR tone rendering [9,29,43], medical imaging enhancement [37], degraded movie color restoration [40]. Several other examples can be found in [28,29].

Among the many Retinex implementations, the respective importance of the threshold and reset mechanisms has been object of a long debate within literature and different variants of the model exist. Thresholding has been proven not to be essential for the Retinex implementation [24] and variants with and without thresholding can be found. Reset is the core of the Retinex model, however variants without the reset can be found, they derive from the latest modifications, proposed by Land in the eighties [19,20], which changed considerably the original model [29]. A rather complete mathematical formalization of the ratio-chain function – encompassing both the thresholding and reset mechanisms – has been given by Provenzi et al. [24].

### 1.1. Scope of the paper

The main focus of [24] is the formalization of the ratio-chain once *both starting point and end point of each path have been chosen* by suitable sampling procedures. The main motivating objective of the present paper is to pass from the sampling-based algorithm, provided in [24] (sampling-based algorithms are intrinsically noisy, due to sampling fluctuations), to a fully analytically computable probabilistic model, based on Markov Chains. In this work, we focus on the development of the probabilistic model of the reset-only (without threshold) version of Retinex.

A few words of clarification are in order at this point. In all the path-based versions of Retinex a walk is charged to carry the influence of a *reference* pixel to the *target* pixel. This influence, as we see formally in the next section, consists in an *influence weight*, which must be computed probabilistically (and translates the prescriptions of the sampling schemas), and in an *influence value*: this, in a resetful and thresholded model, consists in the value of the reference pixel intensity *transformed* according to a recursively defined function of the chain-product, which includes both *thresholding* and *reset* mechanisms.

The thresholding mechanism acts *locally*, at path-step level: it removes the contribution of those steps linking contiguous pixels of similar intensity (it is a *short-range* mechanism). The reset mechanism works potentially at full-path scale: it restarts