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Hesitant fuzzy set lexicographical ordering and its application to multi-attribute decision making

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ABSTRACT

There exist several types of hesitant fuzzy set (HFS) ranking techniques that have been widely used for handling multi-attribute decision making problems with HFS information. The main goal of this paper is to present firstly a brief study of some existing HFS ranking techniques by emphasizing their counterintuitive examples, and then a novel HFS ranking technique is introduced based on the idea of lexicographical ordering.

1. Introduction

Since hesitant fuzzy set (HFS) was introduced originally by Torra [16] to deal with uncertainty, more and more multiple attribute decision making theories and methods with hesitant fuzzy information have been developed [3,12,18,19]. The concept of HFS is an extension of fuzzy set in which the membership degree of a given element, called the hesitant fuzzy element (HFE), is defined as a set of possible values. This situation can be found in a group decision making problem. To clarify the necessity of introducing HFSs, consider a situation in which two decision makers discuss the membership degree of an element x to a set A. One wants to assign 0.2, but the other 0.4. Accordingly, the difficulty of establishing a common membership degree is not because there is a margin of error (as in intuitionistic fuzzy sets [1]), or some possibility distribution values (as in type-2 fuzzy sets [4]), but because there is a set of possible values.

Keeping in mind the concept of HFS, it is difficult sometimes for experts to express the membership degrees of an element to a given set only by crisp values between 0 and 1. To overcome this limitation, different extensions of HFS have been introduced in the literature, such as dual hesitant fuzzy sets (DHFSs) [24], generalized hesitant fuzzy sets (GHFSs) [14], higher order hesitant fuzzy sets (HOHFSs) [9] and hesitant fuzzy linguistic term sets (HFLTSs) [13,15].

As pointed out frequently in the literature, ranking HFEs is significant and plays an indispensable role in the hesitant fuzzy multi-attribute decision making problems.

Up to now, some researchers proposed HFE measuring techniques from different perspectives, and they subsequently extended these techniques to those for HFSs. We may classify HFEs measuring techniques into two main classes with respect to their performance: algorithmic techniques and non-algorithmic techniques. In the algorithmic techniques, the ranking orders determined by performing several steps, for instance, the technique of Chen et al. [3] and that of Liao et al. [12]. The outranking approach proposed by Wang et al. [18] based on traditional ELECTRE methods is another algorithmic technique which is not covered by this study because of its laborious duty. In the non-algorithmic techniques, the ranking order of HFEs is achieved in

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only one step, for instance, the technique of Xia and Xu [19], Xu and Xia's distance measures [20], and Farhadinia's techniques [6,7].

However, as shown later in the next sections, the existing HFSs ranking techniques are not always reasonable and may provide insufficient information on alternatives in some cases. This motivates us to propose a new ranking technique which is not falsified by the counterexamples of existing ones.

An advantage of the proposed HFS ranking method is that the ranking vector associated with HFSs can be easily extended to that associated with DHFSs (as the mean of ranking vectors associated with the possible membership degrees and non-membership degrees) [22]. It is more interesting that, on the basis of the relationship between DHFSs and IFSs [24], the proposed ranking vector can be also associated with IFSs which may be usefully seen as an enrichment of subject of IFS ranking methods [10,11].

The present paper is organized as follows: background on the HFSs and a brief review of some existing HFS ranking techniques underling their counterintuitive examples are given in Section 2. In Section 3, we define, inspired by lexicographical ordering, a novel HFS ranking method and show that the proposed ranking method meets some interesting properties and does not produce inconsistent orderings even if the counterintuitive examples of existing methods are taken into account. In Section 4, we illustrate the applicability of the proposed HFS lexicographic ranking method in multi-attribute decision making problems by means of a practical example. This paper concludes in Section 5.

2. Hesitant fuzzy set and existing HFS ranking techniques

This section is first devoted to describing the basic definitions and notions of fuzzy set (FS) and its new generalization which are referred to as the hesitant fuzzy set (HFS) [16].

An ordinary fuzzy set (FS) A in X is defined [23] as $A = \{\langle x, A(x)\rangle : x \in X\}$, where $A: X \to [0, 1]$ and the real value $A(x)$ represents the degree of membership of x in A.

Definition 2.1 ([19])**.** Let X be the universe of discourse. A hesitant fuzzy set (HFS) on X is symbolized by

$$H = \{\langle x, h(x)\rangle : x \in X\},$$

where $h(x)$, referred to as the hesitant fuzzy element (HFE), is a set of some values in [0, 1] denoting the possible membership degree of the element $x \in X$ to the set H. In this regard, the HFS H can be denoted by

$$H = \{\langle x, \bigcup_{\gamma\in h(x)} \{\gamma\}\rangle : x \in X\}.$$

Example 2.1. If $X = \{x_1, x_2, x_3\}$ is the discourse set, $h(x_1) = \{0.2, 0.4, 0.5\}$, $h(x_2) = \{0.3, 0.4\}$ and $h(x_3) = \{0.3, 0.2, 0.5, 0.6\}$ are the HFEs of x_i $(i = 1, 2, 3)$ to a set H, respectively. Then H can be considered as a HFS, i.e.,

$$H = \{\langle x_1, \{0.2, 0.4, 0.5\}\rangle, \langle x_2, \{0.3, 0.4\}\rangle, \langle x_3, \{0.3, 0.2, 0.5.0.6\}\rangle\}.$$

From a mathematical point of view, a HFS H can be seen as a FS if there is only one element in $h(x)$. In this situation, HFSs include FSs as a special case.

For given three HFEs represented by $h = \bigcup_{\gamma\in h}\{\gamma\}$, $h_1 = \bigcup_{\gamma_1\in h_1}\{\gamma_1\}$ and $h_2 = \bigcup_{\gamma_2\in h_2}\{\gamma_2\}$, the arithmetic operations are defined as follows (see [16,19]):

$$h^\lambda = \bigcup_{\gamma\in h}\{\gamma^\lambda\};$$

$$\lambda h = \bigcup_{\gamma\in h}\{1-(1-\gamma)^\lambda\};$$

$$h_1 \oplus h_2 = \bigcup_{\gamma_1\in h_1, \gamma_2\in h_2} \{\gamma_1+\gamma_2-\gamma_1\gamma_2\};$$

$$h_1 \otimes h_2 = \bigcup_{\gamma_1\in h_1, \gamma_2\in h_2} \{\gamma_1\gamma_2\}.$$

In what follows, we will describe briefly the existing techniques which are available for ranking HFEs. Farhadinia [7] showed that a ranking function of HFS is directly defined by the use of ranking function of its HFEs. Therefore, we mainly discuss here the ranking functions for HFEs, and drop the discussion on the corresponding ranking functions for HFSs.

Hereafter, for notational convenience, h stands for the HFE $h(x)$ for $x \in X$, and we assume that $|h| = n$, that is,

$$h = \bigcup_{\gamma\in h}\{\gamma\} = \{\gamma^{(1)}, \gamma^{(2)}, \ldots, \gamma^{(n)}\},$$

where all the elements in h are arranged in increasing order.

Definition 2.2 ([8])**.** Let $h_1 = \bigcup_{\gamma\in h_1}\{\gamma\} = \{\gamma_1^{(1)}, \gamma_1^{(2)}, \ldots, \gamma_1^{(n)}\}$ and $h_2 = \bigcup_{\gamma\in h_2}\{\gamma\} = \{\gamma_2^{(1)}, \gamma_2^{(2)}, \ldots, \gamma_2^{(n)}\}$ be two HFEs. The component-wise ordering of HFSs is defined as

$$h_1 \preceq h_2 \quad \text{if and only if} \quad \gamma_1^{(i)} \le \gamma_2^{(i)}, \quad 1 \le i \le n. \tag{1}$$