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Observability analysis of inertial navigation errors from optical flow subspace constraint

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ABSTRACT

Fusion of inertial and vision sensors is an effective aid to inertial navigation systems (INS) during GPS outage. Optical flow-aided inertial navigation circumvents feature tracking, landmark mapping, and state vector augmentation typical of simultaneous localization and mapping (SLAM). This paper focuses on the observability analysis of INS errors from implicit measurements of the optical flow subspace constraint, and derives how observable and unobservable directions are affected by the motion of a camera rigidly coupled to an inertial measurement unit (IMU). Straight motion and piecewise constant (PWC) attitude segments yield the random constant IMU errors observable. The unobservable directions are the three-dimensional (3D) position error, the velocity error along the ground velocity, and the combination of angular misalignment about the local vertical and the velocity error along the horizontal direction orthogonal to the ground velocity. The velocity error along the ground velocity becomes observable with horizontal maneuvering. A Monte Carlo simulation validates the observability analysis, and reveals the feasibility of IMU calibration and the mitigation of navigation error growth with the aid of the optical flow subspace constraint compared with the unaided INS.

1. Introduction

The observability analysis of linear inertial navigation systems (INS) error models in INS/GPS integration has shown the beneficial effect of maneuvering on the estimation uncertainty of attitude misalignment and inertial measurement unit (IMU) errors during in-flight alignment and the corresponding enhancement of navigation accuracy [15,16,51,38,24,48]. More recently, the awareness that low-cost receivers in INS/GPS integration can be vulnerable to jamming has further motivated the research of INS/vision integration for the estimation of IMU errors [46,27].

Fusion of inertial and vision sensing combined with *a priori* information on the location of landmarks improves simultaneous localization and mapping (SLAM) [25,8]. The computational complexity of SLAM, however, increases with the augmentation of the state vector with information related to the viewed landmarks. Instances of such information when tracking image

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Nomenclature

B	body
CAM	camera
DCM	direction cosine matrix
e, ECEF	earth-fixed earth-centered
i, I	inertial
im	image
IEKF	implicit extended Kalman filter
IMU	inertial measurement unit
INS	inertial navigation system
LOM	local observability matrix of the linear time-varying dynamics system
NED	North, East, Down
PWC	piece-wise constant
SOM	stripped observability matrix of the linear PWC dynamics system
TOM	total observability matrix of the linear PWC dynamics system
wrt	with respect to
3D	three-dimensional

Notation

$\mathbf{A_{sp}}$	specific force vector
$\mathbf{A_x}$	vector $\mathbf{A}$ represented in coordinate frame S_x
$\mathbf{A_{x,m}}$	measured vector $\mathbf{A}$ represented in coordinate frame S_x
$\mathbf{A_t}$	short notation for the time-varying specific force vector $\mathbf{A_{sp}}$ represented in coordinate frame S_{NED} [see $\mathbf{A_t}=\mathbf{A_{sp,NED}}(t)$ in Section 3.2]
$\mathbf{A^T}$	vector or matrix $\mathbf{A}$ transposed
$\hat{\mathbf{A}}$	estimate of vector or matrix $\mathbf{A}$
$\mathbf{A}^{+}$	pseudoinverse of matrix $\mathbf{A}$
$\mathbf{A}^{\perp}$	orthogonal projector on the orthogonal complement of the range space of matrix $\mathbf{A}$
$[\mathbf{A}\times]$	skew-symmetric matrix representation of the vector cross product $\mathbf{A}\times(.)$
c_x, c_z	image center (principal point) coordinates in the image coordinate frame S_{im}
$\mathbf{D}_y^x$	DCM rotating a vector representation from coordinate frame S_x to S_y
$\mathbf{D_t}$	short notation for the time-varying specific force DCM rotating a vector representation from S_{CAM} to S_{NED} [see $\mathbf{D_t}=\mathbf{D}^{\mathbf{CAM}}_{\mathbf{NED}}(t)$ in Section 3.2]
f	focal length
$\mathbf{F_i}$	INS error state dynamics matrix in the ith PWC camera attitude segment
g, $\mathbf{g}$	local gravity magnitude and vector, respectively
$\mathbf{H_i}$	linearized subspace constraint measurement matrix in the ith PWC camera attitude segment
$\mathbf{H_k}$	linearized subspace constraint measurement matrix in the kth sampling time t_k of the horizontal maneuver
$\mathbf{L}$	Jacobian [see (13)] of the optical flow subspace constraint $\mathbf{C}^{\perp}\dot{\boldsymbol{\eta}}=\mathbf{0}$ [see (6)] with respect to the ground velocity $\mathbf{V_{CAM}}$.
N	number of detected features in the image plane giving rise to optical flow
$\mathbf{O}(k)$	local observability matrix in the kth sampling time t_k as of initial time t_0.
R, $\mathbf{R}$	distance from the Earth center and position vector, respectively
S_x	x coordinate frame, $x=B$, CAM, i, im, ECEF, NED etc.
$\mathbf{Q}(r)$	Total observability matrix after r PWC camera attitude segments
t_k	kth sampling time as of the onset of a horizontal maneuver [see Section 3.2]
$\mathbf{V}$,$\mathbf{V_{CAM}}$	camera egomotion ground velocity wrt the static scene
$\hat{\mathbf{V}}^{\perp}$	unit vector orthogonal to both vectors $\mathbf{g}$ and **V:** $\hat{\mathbf{V}}^{\perp}=[\mathbf{g}\times]\mathbf{V}/(\lvert\mathbf{g}\rvert\lvert\mathbf{V}\rvert)$
$X_{\text{CAM},i}$	depth of the ith detected feature in the image plane in S_{CAM} coordinates, $i=1,\ldots,N$
yi, zi	position of the ith detected feature in the image plane in the S_{im} coordinate frame, $i=1,\ldots,N$
Δ_i	duration of the ith PWC camera attitude segment
$\boldsymbol{\Delta}\mathbf{R}$	INS position error vector
$\boldsymbol{\Delta}\mathbf{V}$	INS ground velocity error vector
$\boldsymbol{\Delta}\mathbf{x}$	INS error state vector $\in\Re^{15}$
$\boldsymbol{\varepsilon}$	vector of drifts in the IMU rate-gyro triad
$\boldsymbol{\psi}$	attitude misalignment vector from the S_{NED} at the INS-computed position to the analytical platform S_{NED} computed with data from the IMU rate-gyros
$\boldsymbol{\nabla}$	vector of biases in the IMU accelerometer triad