



Distance-based consensus models for fuzzy and multiplicative preference relations



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ABSTRACT

This paper proposes a distance-based consensus model for fuzzy preference relations where the weights of fuzzy preference relations are automatically determined. Two indices, an individual to group consensus index (*ICI*) and a group consensus index (*GCI*), are introduced. An iterative consensus reaching algorithm is presented and the process terminates until both the *ICI* and *GCI* are controlled within predefined thresholds. The model and algorithm are then extended to handle multiplicative preference relations. Finally, two examples are illustrated and comparative analyses demonstrate the effectiveness of the proposed methods.

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1. Introduction

Group decision making (GDM) is concerned with deriving a solution from a group of independent decision-makers' (DMs') heterogeneous preferences over a set of alternatives. Before the final choice is identified, two processes are usually carried out: (1) a consensus process and (2) a selection process. The first process addresses how to obtain a maximum degree of consensus or agreement among the DMs over the alternative set, while the second process handles the derivation of the alternative set based on the DMs' individual judgment on alternatives [24].

Numerous approaches have been put forward for consensus measures based on different types of preference relations, including consensus models for ordinal preference [14–16,19], linguistic preference relations [3,4,7–10,17,26–28,58], multi-attribute GDM problems [5,20,21,37,50,59], intuitionistic multiplicative preference relations [29], and other preference relations [1,24,35,38].

The consensus reaching process has been widely studied for multiplicative preference relations (MPRs). Van den Honert [45] proposed a model to represent a consensus-seeking GDM process based on the analytic hierarchy process (AHP) framework, where group preference intensity judgments are expressed as random variables with associated probability distributions. Dong et al. [18] developed AHP consensus models by using a row geometric mean prioritization method. Wu and Xu [48] presented a consistency and consensus-based model for GDM with MPRs. Gong et al. [22] developed a group consensus deviation degree optimization model for MPRs that minimizes the weighted arithmetic mean of individual consistency deviation degrees. Xu [60] put forward a consensus reaching process for GDM with incomplete MPRs.

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For fuzzy preference relations (FPRs), Kacprzyk and Fedrizzi [30] devised a ‘soft’ measure of consensus. Chiclana et al. [12] furnished a framework for integrating individual consistency into a consensus model. The paradigm consists of two processes: an individual consistency control process and a consensus reaching process. Based on this work, Zhang et al. [67] proposed a set of linear optimization models to address certain consistency issues on FPRs, such as individual consistency construction, consensus modeling and management of incomplete fuzzy preference relations. Herrera-Viedma et al. [23] presented a new consensus model for GDM problems with incomplete fuzzy preference relations. The key feature is to introduce a feedback mechanism for advising DMs to change or complete their preferences so that a solution with high consensus and consistency degrees can be reached. Parreiras et al. [36] proposed a dynamical consensus scheme based on a nonreciprocal fuzzy preference relation modeling. Wu and Xu [46] developed a consistency consensus based decision support model for GDM. Recently, Xu and Cai [62] put forth a number of goal programming and quadratic programming models to maximize group consensus. The main purpose is to determine importance weights for FPRs and MPRs. However, as pointed out in Section 2, a significant drawback exists for their quadratic programming models as the derived weight is always the same for each expert. Furthermore, for existing consensus models for improving consensus indices, it is often the case that the final improved preference relations significantly differ from the DMs’ original judgment information, as testified by examples in [1,3–10,12,17,18,20–23,26–28,46–50,59,60,62,67,68]. It is the authors’ belief that GDM should utilize the DMs’ opinions on the alternatives to find a solution. If DMs’ opinions are significantly distorted, the derived solution is likely questionable. In order to obtain a reliable solution, the decision model should retain the DMs’ opinions as much as possible. To address these deficiencies, a new consensus measure should be designed to make use of group judgments.

This paper first puts forward a distance-based consensus model for FPRs to derive each DM’s individual weight vector, then an aggregation operator is developed to obtain a collective FPR. An individual to group consensus index (ICI) and a group consensus index (GCI) are subsequently introduced, followed by an iterative algorithm for consensus reaching with a stoppage condition when both ICI and GCI are lower than predefined thresholds. The model and algorithm are then extended to MPRs.

The remainder of this paper is organized as follows. Section 2 briefly reviews group consensus models introduced by Xu and Cai [62] for FPRs with comments on their drawbacks. Section 3 develops a distance-based model to determine DMs’ weights for GDM with FPRs, and puts forward an algorithm for the consensus reaching process. Section 4 extends the model and algorithm to solve consensus problems with MPRs. In Section 5, two illustrative examples are developed and the results are compared with those obtained with existing approaches. Concluding remarks are furnished in Section 6.

2. A review of group consensus based on fuzzy preference relations

For a GDM problem, let $X = \{x_1, x_2, \dots, x_n\}$ ($n \geq 2$) be a finite set of alternatives and $E = \{e_1, e_2, \dots, e_m\}$ ($m \geq 2$) be a finite set of DMs. In a multi-criteria decision making problem, a DM e_k often compares each pair of alternatives in X and provides his/her preference degree $p_{ij,k}$ of alternative x_i over x_j on a 0–1 scale, where $0 \leq p_{ij,k} \leq 1$, $p_{ij,k} = 0.5$ denotes e_k ’s indifference between x_i and x_j , $p_{ij,k} = 1$ denotes that x_i is definitely preferred to x_j by e_k , and $0.5 < p_{ij,k} < 1$ (or $0 < p_{ji,k} < 0.5$) denotes that x_i is preferred to x_j by e_k with a varying degree of likelihood. All preference values $p_{ij,k}$ ($i, j = 1, 2, \dots, n$) provided by DM e_k are denoted as an FPR $P_k = (p_{ij,k})_{n \times n}$ [11,25,31,33,40–44,46,51–57]

$$0 \leq p_{ij,k} \leq 1, \quad p_{ii,k} = 0.5, \quad p_{ij,k} + p_{ji,k} = 1, \quad i, j = 1, 2, \dots, n \tag{1}$$

In a GDM problem, let $w = (w_1, w_2, \dots, w_m)^T$ be the unknown weight vector for FPRs $P_k = (p_{ij,k})_{n \times n}$ ($k = 1, 2, \dots, m$), where

$$\sum_{k=1}^m w_k = 1, \quad w_k \geq 0, \quad k = 1, 2, \dots, m \tag{2}$$

To obtain a collective judgment for the group, Xu and Cai [62] employed the Weighted Arithmetic Averaging (WAA) operator:

$$p_{ij} = \sum_{k=1}^m w_k p_{ij,k}, \quad i, j = 1, 2, \dots, n \tag{3}$$

to aggregate individual FPRs $P_k = (p_{ij,k})_{n \times n}$ ($k = 1, 2, \dots, m$) into a collective preference relation $P = (p_{ij})_{n \times n}$. It can be easily shown that P satisfies condition (1), and is thus also an FPR.

Clearly, a key issue in applying the WAA operator is to determine the weight vector w . If an individual FPR P_k is consistent with the collective FPR P , then $P_k = P$, i.e., $p_{ij,k} = p_{ij}$, for all $i, j = 1, 2, \dots, n$. Using (3), we have

$$p_{ij,k} = \sum_{l=1}^m w_l p_{ij,l}, \quad \text{for all } i, j = 1, 2, \dots, n \tag{4}$$

However, generally speaking, Eq. (4) does not always hold. Let

$$\varepsilon_{ij,k} = \left| p_{ij,k} - \sum_{l=1}^m w_l p_{ij,l} \right|, \quad \text{for all } i, j = 1, 2, \dots, n, \quad k = 1, 2, \dots, m \tag{5}$$

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