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Input-to-state stability for discrete-time nonlinear switched singular systems

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ABSTRACT

This paper investigates the input-to-state stability (ISS) problems for a class of discrete-time nonlinear switched singular systems (SSSs). Two novel ISS criteria are proposed based on average dwell time (ADT) approach and iterative algorithm of discrete-time systems (IADS). In particular, the following two cases are considered for the underlying systems: the first case is that all the sub-systems are exponentially stable, and the other case is that only a few sub-systems are exponentially stable. The corresponding ISS criteria are obtained to guarantee that the closed-loop systems are input-to-state stable via ADT approach and IADS. We neither construct a specific Lyapunov function for ISS in the proof process of the stability criteria nor design a specific structure of the control inputs. The design deficit of the switching controllers is optimized and the design difficulty of the switching controllers is reduced via the proposed criteria. Finally, two numerical examples are provided to illustrate the feasibility of the results obtained.

1. Introduction

Singular systems provide a natural representation for modeling of dynamic systems, and conveniently describe a great number of physical systems better than standard state-space systems. They are also referred as descriptor systems, implicit systems, semi-state systems or differential-algebraic equations. So far, they have extensive potential applications in real world, such as electrical circuits, constraint robots, and mechanical systems [8]. Some of fundamental notions and results on the theory of standard state-space systems have been extended to the areas of singular systems. For instance, the problems on stability, detection, controller, observer and filter design for singular systems have been studied in [4,6,12,13,22,23,34,38,39,43]. On the other hand, switched systems are an important class of hybrid dynamic systems, in which switching controllers play a nontrivial role. They are often used to exhibit many physical phenomena or practical applications displaying switching features. They have gained considerable interest in both theoretical researches and practical applications, such as power electronics, chaos generators, computer disc drivers and intercepting missiles [14,15,26]. Such dynamic systems admit continuous signals that take values from a vector space and discrete signals that take values from a discrete index set [29]. It is for this very reason that switched dynamic systems have received considerable attention in automatic control areas and some recent works focus on switched systems have emerged [21,27,28,30,36,37,47].

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Switched singular systems (SSSs) have been developed from a class of switched systems when partial or total sub-systems of switched systems have a certain singular perturbation [25]. SSSs are ubiquitous and arise in a great deal of different application areas, such as boost or buck converters [9] and coupled networks [41]. Until now, both analysis and synthesis for SSSs are still difficult but some research achievements on such systems have been investigated, such as admissibility via transition rate matrix quantized approach [32] and stable-protocol admissibility via orthonormal projection operator [44]. In addition, some important results on the design of controllers for SSSs have been reported, such as robust control [31], sliding mode control [35] and reliable dissipative control [17]. It is become more and more important and necessary to make further study for SSSs. So far, some results focus on the stability of continuous-time SSSs, for instance [33,40,46], but quite a few works focus on the stabilization of discrete-time SSSs, for instance [16]. Therefore, SSSs, on which studies are still open and unsolved, should be attached more attention in automatic control areas.

Input-to-state stability (ISS) aims to investigate how external disturbance inputs affect the system stability and plays an important role in analysis and synthesis of nonlinear systems [11,18]. Deterministic system states can return to their inherent equilibrium states for arbitrary initial conditions, and Lyapunov stability theory mainly devotes to deterministic systems without external perturbations. It is not suitable for dealing with nonlinear systems with perturbations when it is required to determine whether their responses are regulated in a desired region or not [48]. ISS indicates that deterministic systems are asymptotically stable in the Lyapunov sense and the responses with control inputs are bounded. Compared with Lyapunov stability, ISS for nonlinear systems are more in line with the needs of practical applications. The researches on ISS of dynamic systems with perturbations have quickly become one of the active research topics in automatic control areas. It has been widely employed in stability analysis and controller designs of nonlinear systems. The ISS problems of impulsive switched systems with time-delay and without time-delay are concerned in [19,20]. The ISS criteria of impulsive stochastic systems are proposed in [45]. The ISS properties of switched systems by average dwell-time switching signals are addressed in [5]. The ISS problems for switched Hopfield neural networks with time-delay are presented in [1]. The ISS criteria for digital filters with external interference are proposed via linear matrix inequality and the effect of external interference can be attenuated by ISS properties in [3]. Semiglobal practical integral ISS properties of a class of parameterized discrete-time interconnected systems are proposed in [24]. Razumikhin-type theorems are established to guarantee exponential ISS and integral asymptotic ISS for time-delay impulsive nonlinear systems in [7]. The ISS properties of constrained nonlinear systems with unknown but bounded disturbance are presented and nonlinear H_∞ model predictive control are designed in [10]. Nonlinear filtering can be devised for a class of continuous nonlinear systems with time-delays by ISS properties in [2]. The stochastic ISS problems of switched stochastic nonlinear systems have been concerned in [49]. Discrete-time nonlinear SSSs are so complicated that the researches on such systems have been seldom investigated and this motivates us for this study.

It is commonly recognized that discrete-time singular systems are more and more complicated in stability analysis and controller designs compared with standard state-space systems. In general, discrete-time singular systems have three types of modes: finite dynamic modes, causal modes and non-dynamic modes, however, the latter two kinds of modes are not included in standard state-space systems. Therefore, causality, regularity and stability should be considered simultaneously. The stability analysis and controller designs of switched systems have the following basic problems: search for stability conditions under arbitrary switching, identify the limited but useful class of switching signals and construct stabilizing switching signals. We deem from the above that the ISS problems for nonlinear SSSs belong to an important class of research orientations in theoretical and practical considerations. The ISS problems of continuous nonlinear SSSs are concerned and some sufficient conditions are presented to guarantee that the whole system is input-to-state stable via appropriate dwell time switching rules in [9]. However, up to date and to the best of our knowledge, the researches on ISS of discrete-time nonlinear SSSs have been seldom concerned via the dwell time approach. It is necessary to promote us to carry out this work meticulously in this paper.

Inspired by the discussion above, in this paper, we will investigate the ISS problems for a class of discrete-time nonlinear SSSs. As far as we know, the switching rules are much crucial for nonlinear SSSs. Even if all the sub-systems of discrete-time nonlinear SSSs are exponentially stable, the whole system is still unstable possibly. There are two cases on the ISS problems of discrete-time nonlinear SSSs. The first case is that all the sub-systems are exponentially stable and the other case is that only partial sub-systems are exponentially stable. The ISS criteria for the underlying SSSs are proposed to ensure that the closed-loop systems are input-to-state stable via average dwell time (ADT) approach and iterative algorithm of discrete-time systems (IADS). It is much simple to design each sub-controller of switching controllers. Furthermore, numerical examples and their extensive simulations are provided to illustrate the feasibility and effectiveness of the results obtained.

The remaining of this paper is organized as follows. Some of relevant definitions and preliminaries on ISS for SSSs are briefly sketched in Section 2. The two cases that all the slow sub-systems of SSSs are exponentially stable and only partial sub-systems are exponentially stable are concerned. Sufficient conditions are derived to ensure that the closed-loop systems are input-to-state stable based on ADT approach and IADS in Section 3. In Section 4, two numerical examples are presented to illustrate the feasibility and validity of the results obtained. Section 5 includes some concluding remarks.

Notations. For a real symmetric matrix $\boldsymbol{A}$, $\boldsymbol{A} > 0(\boldsymbol{A} \geq 0)$ represents that it is positive definite (semi-positive definite). $\mathcal{Z}^+$ denotes the set of all nonnegative integers. $\mathcal{R}$ is the set of all real numbers, and $\mathcal{R}^+$ refers to the subset of nonnegative elements of $\mathcal{R}$, defined by $\mathcal{R}^+ = [0, +\infty)$. $\mathcal{R}^n$ is the n dimensional real Euclidean space and $\mathcal{R}^{m\times n}$ is the set of all $m \times n$ real matrices. The superscript "T" means the transpose. $\|\cdot\|$ stands for the Euclidean norm for a given vector or matrix. diag{...} is a block-diagonal matrix.