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Latticized linear programming subject to max-product fuzzy relation inequalities with application in wireless communication[☆]

Xiao-Peng Yang [a,b], Xue-Gang Zhou [b,c], Bing-Yuan Cao [b,d,*]

[a] School of Mathematics and Statistics, Hanshan Normal University, Chaozhou, Guangdong 521041, China
[b] School of Mathematics and Information Science, Guangzhou University, Guangzhou, Guangdong 510006, China
[c] Department of Applied Mathematics, Guangdong University of Finance, Guangzhou, Guangdong 510521, China
[d] Guangzhou Vocational College of Science and Technology, Guangzhou, Guangdong 510550, China

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ABSTRACT

In this paper we introduce the latticized linear programming problem subject to max-product fuzzy relation inequalities with application in the optimization management model of wireless communication emission base stations. Resolution of max-product fuzzy relation inequalities is studied by comparing with that of the corresponding max-product fuzzy relation equations. A solution matrix approach is developed for solving the proposed problem without finding all the (quasi-) minimal solutions of the constraint. For carrying out the solution matrix approach, we provide a step-by-step algorithm illustrated by a numerical example.

1. Introduction

Fuzzy relation equations (FRE) may be written in its matrix form, i.e.

$$A \circ x^T = b^T \tag{1}$$

where $A = (a_{ij})_{m\times n} \in [0,1]^{m\times n}$, $x = (x_1, x_2, \ldots, x_n) \in [0,1]^n$, $b = (b_1, b_2, \ldots, b_m) \in [0,1]^m$, and $\circ$ represents a composition operator. Resolution of fuzzy relation equations with max-min composition (max-min fuzzy relation equations) was first studied by Sanchez [30]. Besides, Sanchez [31] developed the application of FRE in medical diagnosis in biotechnology. Since then the composition operator in fuzzy relation equations was replaced by max-product and furthermore extended to the general max-t-norm composition operator [5,8,24,28]. The resolution method was kept in improving. In fact, if the max-t-norm fuzzy relation equations is consistent (solvable), then its solution set is completely determined by a unique maximum solution and a finite number of minimal solutions. FRE was applied in various fields [6,7,20,25,26]. Although the FRE with

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* Corresponding author at: School of Mathematics and Information Science, Guangzhou University, Guangzhou, Guangdong 510006, China. Tel.: +86 15876504405.

E-mail address: caobingy@163.com (B.-Y. Cao).

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max-min operator was widely applied, the max-product operator was superior to the max-min operator in many specific cases [10,27,38]. For the resolution of max-product FRE, the readers might refer to [3,19,22,29,32,33,36].

Most of the existing literatures in this research field focused on the resolution of FRE and its relevant optimization problems. However, only a few research works investigated the fuzzy relation inequalities (FRI) and its relevant optimization problems.

Wang et al. [35] studied the properties of max-min FRI, based on which an effective algorithm was developed to deal with the corresponding fuzzy relation latticized optimization problem. In [35], the conservative path method was proposed to find out all the minimal solutions of the max-min FRI. Subsequently the optimal solutions were selected from the minimal solutions by pairwise comparison. The latticized linear programming problem subject max-min FRI was also investigated in the works [16,17]. Li and Fang [16] obtained an optimal solution of the latticized linear programming problem without finding all the minimal solutions. Besides they studied some variants of the problem. In [17], based on the concept of semi-tensor product, a matrix approach was applied to handle the latticized linear programming problem subject max-min FRI. Guo et al. [12] studied the general linear programming problem with max-min FRI constraint.

In recent years, Drewniak and Matusiewicz were interested in max-$*$ fuzzy relation equations and inequalities with the increasing operation $*$ continuous on the second argument [9,23]. They investigated some useful properties and developed a novel algorithm to find the set of all minimal solutions. As shown in [9], both max-min and max-product were members of max-$*$ composition operators. Interestingly, Li and Yang [18] introduced the so-called addition-min fuzzy relation inequalities to characterize a peer-to-peer file sharing system. It is obvious that addition-min is not a max-$*$ composition operator. Furthermore, based on the concept of pseudo-minimal index, Yang [39] developed a pseudo-minimal-index algorithm to minimize a linear objective function with addition-min fuzzy relation inequalities constraint. To improve the results presented in [39], Yang et al. [44] proposed the min-max programming subject to addition-min fuzzy relation inequalities. By some novel techniques, the primary problem was converted into several subproblems and solved. They also studied the multi-level linear programming problem with addition-min fuzzy relation inequalities constraint [42].

The optimization problems with general nonlinear objective functions and fuzzy relation equations or inequalities constraints were studied in [13–15,21]. In general, the genetic algorithm was applied to deal with this kind of problems. However, some fuzzy relation nonlinear optimization problems could be solved by some specific method. Abbasi Molai [1,2] focused on the fuzzy linear and quadratic optimization problems subject to max-product fuzzy relation inequalities. Based on the set of quasi-minimal solutions, some effective resolution methods were proposed to deal with the fuzzy relation quadratic programming problem in [1]. Fuzzy relation geometric programming problem was introduced by Yang and Cao [4,40,41]. Some variants of the fuzzy relation geometric programming problem were investigated later in [34,37]. The geometric programming objective function was decomposed into two subfunctions according to the coefficients. Correspondingly, the main problem was decomposed into two subproblems. One of the subproblems could be easily solved since the unique maximum solution of the feasible domain was exactly its optimal solution. To deal with the other subproblem, the authors found its optimal solution(s) from the minimal solutions (or quasi-minimal solutions) of the feasible domain. Recently, as an extension of the previous works on fuzzy relation geometric programming, Yang et al. [43] studied the single-variable term semi-latticized geometric programming subject to max-product fuzzy relation equations. The proposed problem was devised from the peer-to-peer network system and the target was to minimize the biggest dissatisfaction degrees of the terminals in such system.

In this paper, due to the practical application background (see Section 2), we are interested in the latticized linear programming subject to max-product fuzzy relation inequalities. The rest of the paper is organized as follows. In Section 2 we show the application background of the latticized linear programming subject to max-product FRI. In Section 3, resolution of max-product FRI is studied by comparison with max-product FRE. A solution matrix approach for solving the proposed problem is introduced in Section 4 with a step-by-step algorithm. Section 5 provides a numerical example to illustrate the algorithm. Simple advantages of our solution method and conclusions are given in Section 6 and 7, respectively.

2. Problem statement

Nowadays wireless communication is widely applied in various fields, such as mobile communication and information transmission. In this paper we consider a kind of wireless communication optimization management models. In such wireless communication model, the information is transmitted by the electromagnetic wave, while the electromagnetic wave is emitted from some fixed emission base stations (EBSs). Our target is to optimize the radiation intensity of electromagnetic wave emitted from the fixed emission base station (EBS). As we know, high radiation intensity will ensure good communication quality, but meanwhile, it will damage the health of humans. Next we establish the optimization model in the wireless communication EBS operation system. Assume that there are n EBSs, i.e. $A_1, A_2, \ldots, A_n$, located in different places of a specific area, such as a city. The jth EBS will emit electromagnetic wave with radiation intensity $x_j > 0$, $j = 1, 2, \ldots, n$. The communication quality level is determined by the intensity of electromagnetic radiation. In order to satisfy the requirement of communication quality level, m testing points, i.e. $B_1, B_2, \ldots, B_m$, are selected to test the intensity of electromagnetic radiation. For example, when the wireless communication is applied in the cell phone network, the testing point is usually the place with higher population density. At the ith testing point B_i, the intensity of electromagnetic radiation emitted from A_j, denoted by r_{ij}, will belong to $[0, x_j]$, $i \in I = \{1, 2, \ldots, m\}$, $j \in J = \{1, 2, \ldots, n\}$. Here, I and J are two index sets. In fact, r_{ij} is related to the distance between B_i and A_j. Furthermore, there exists a real number $a_{ij} \in [0, 1]$ such that $r_{ij} = a_{ij}x_j$.