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A parallel primal-dual splitting method for image restoration

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ABSTRACT

We develop a parallel primal-dual splitting method to solve large-scale image restoration problems, which involve the sum of several linear-operator-coupled nonsmooth but proximable terms. With the proposed method, the objective function is decomposed into pieces that can be processed individually. No inverse operator is involved in our method and the highly parallel structure makes it preferable in distributed computation. The convergence is proven and the convergence rate is analyzed. Besides, its equivalence to the relaxed parallel linearized alternating direction method of multipliers (PLADMM) is addressed. Applications to image restoration problems with compound l_1-regularizer and comparisons with state-of-the-art methods are detailed to show the superiority of the proposed method.

1. Introduction

1.1. Problem formulation

Image and video degradations frequently arise in acquisition and transmission. Some degradations, such as compressed sensing [4], are intentional and beneficial, whereas, the others are annoying and troublesome. However, they all need to be undone for further image processing tasks. In general, the degradation process of an image can be modeled as

$$\boldsymbol{f} = \boldsymbol{K}u + \boldsymbol{n}, \tag{1}$$

where $\boldsymbol{u}, \boldsymbol{f} \in \mathbb{R}^{mno}$ are the original image and the observed image, respectively, which both possess an $m \times n \times o$ domain (o-channel) that is expressed in vector form; $\boldsymbol{K} \in \mathbb{R}^{mno} \times \mathbb{R}^{mno}$ is the ill-posed matrix operator that models the acquisition processing; and $\boldsymbol{n} \in \mathbb{R}^{mno}$ is a vector of some type of additive noise. Due to the ill-posed $\boldsymbol{K}$, the estimation of $\boldsymbol{u}$ from $\boldsymbol{f}$ is an ill-posed linear inverse problem (IPLIP) and the solution of (1) is highly sensitive to the noise or the perturbation in $\boldsymbol{f}$.

Let $D(\boldsymbol{Ku}, \boldsymbol{f})$ be the data-fidelity term and $J(\boldsymbol{u})$ be the regularizer, which often incorporates some sorts of prior knowledge of the original image, e.g., sparsity [14,26,27], low-rank [29,31], and smoothness [3]. Then most approaches to IPLIPs in imaging, such as deblurring [8,24], super-resolution [38], inpainting [17], and segmentation [25,37], result in the following regularized minimization functional form

$$\min_{\boldsymbol{u}} J(\boldsymbol{u}) + \lambda D(\boldsymbol{K}u, \boldsymbol{f}), \tag{2}$$

or its equivalent constrained form

$$\min_{\boldsymbol{u}} J(\boldsymbol{u}) \quad \text{s.t.} \quad D(\boldsymbol{K}u, \boldsymbol{f}) \le c, \tag{3}$$

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where λ is the regularization parameter that balances the data-fidelity and the regularizer; c is a noise-dependent constant upper bound. Due to an intractable data-fidelity constraint, most studies prefer (3) to (2); however, a person should manually attempt many iterations to select an approximate optimal λ. Objective (2) is more attractive if c can be rationally estimated based on the noise level, as λ can be adaptively estimated. Some properties, such as the nonsmoothness, often make the regularizer difficult to be jointly optimized with the data-fidelity part [11]. Besides, the combination of several regularizers may encourage the solution to have more attractive properties; however, this leads to more complex optimization problems.

In this paper, we consider the solution of a class of large-scale image restoration problems that involves a linear combination of several regularizers. They usually lead to the following convex optimization objective

$$\min_{\boldsymbol{x}\in X} g(\boldsymbol{x}) + \sum_{h=1}^{H} f_h(\boldsymbol{L}_h\boldsymbol{x}). \tag{4}$$

Denote the set of all convex, proper, and lower semicontinuous functions [2] from the Hilbert space X to $\mathbb{R}\cup\{+\infty\}$ as $\Gamma_0\{X\}$. In (4), $g \in \Gamma_0\{X\}$ and $f_h \in \Gamma_0\{V_h\}$ are convex functions that are “simple” enough as their proximity operators (defined by (6)) possess closed-form representations or can be efficiently solved. Every $\boldsymbol{L}_h(X \to V_h)$ is a bounded linear operator with the adjoint $\boldsymbol{L}_h^*$ and the induced norm $\|\boldsymbol{L}_h\| = \sup\{\|\boldsymbol{L}_h\boldsymbol{x}\|_2 : \|\boldsymbol{x}\|_2 = 1\} < +\infty$. We assume that the set of minimizers of (4) is nonempty. Given a nonempty closed convex set Ω in X, its indicator function is defined as $\iota_\Omega(\boldsymbol{x})$ ($\iota_\Omega(\boldsymbol{x}) = 0$ if $\boldsymbol{x} \in \Omega$, otherwise, $\iota_\Omega(\boldsymbol{x}) = +\infty$). With ι_Ω, the problem of minimizing $g \in \Gamma_0\{X\}$ over Ω can be recast as $\min_{\boldsymbol{x}\in X} g(\boldsymbol{x}) + \iota_\Omega(\boldsymbol{x})$. Therefore, formulation (4) is sufficiently general for solving many inverse problems, such as (2) and (3) in a broad spectrum of areas.

The set-valued subdifferential [2] of g is defined by

$$\partial g(\boldsymbol{x}) = \{\boldsymbol{b} \in X \,|\, (\forall \boldsymbol{x}' \in X)\ \langle \boldsymbol{x}' - \boldsymbol{x}, \boldsymbol{b}\rangle + g(\boldsymbol{x}) \le g(\boldsymbol{x}')\}. \tag{5}$$

The proximity operator of g is defined by

$$\mathrm{prox}_g : X \to X, \boldsymbol{x} \to \arg\min_{\boldsymbol{x}'\in X} g(\boldsymbol{x}') + \frac{1}{2}\|\boldsymbol{x} - \boldsymbol{x}'\|_2^2. \tag{6}$$

For an indicator function, its proximity operator is simply the projection operator onto the set in which it is defined [2]. The conjugate of g is defined by $g^*(\boldsymbol{x}) = \sup_{\boldsymbol{x}'\in X}\{\langle \boldsymbol{x}, \boldsymbol{x}'\rangle - g(\boldsymbol{x}')\}$.

The major difficulty in solving (4) stems from two aspects. First, the data spaces X and V_h in practical applications are typically of high dimension. Second, the function g and the linear operator coupled f_h may be nondifferentiable, which often makes the operator splitting approach [2] the only viable way to solve (4) [11].

1.2. Related studies

Many operator splitting methods have been proposed to solve the IPLIP, including the forward-backward splitting (FBS) method [2], the Douglas-Rachford splitting (DRS) method [21], the alternating direction method of multipliers (ADMM) [1], the Bregman splitting (BS) method [16], the linearized ADMM (LADMM) [31], and the primal-dual splitting (PDS) method [5,9,10]. The DRS method, the ADMM, and the BS method are equivalent under linear constraints [32]. The PDS method can simultaneously find the solution of the primal problem and its dual counterpart. The convergence rate analysis [6,12,21] of these splitting methods and their generalization to parallel splitting [13,28] are important research cutting edges. The parallel splitting method can solve more complicated problems such as (4).

Generally, splitting methods dealing with the IPLIP involve linear inverse operation. Condat [10,11] proposed a PDS method to solve the convex minimization problem with the form (4). With the exclusion of the linear inverse operation that commonly exists in methods dealing with IPLIP, it solves the primal problem (4) and its dual problem jointly by finding a saddle point of the Lagrangian of (4). However, all functional terms are treated equally in the PDS scheme of Condat, though different terms may represent different physical senses in real-world applications. This may weaken the capability (especially in speed) of the primal-dual scheme.

1.3. Motivation and contributions

In this paper, we generalize the idea of the PDS method [11] and propose a parallel primal-dual splitting (PPDS) method. We prove that the proposed method is equivalent to the parallel LADMM (PLADMM) with relaxation. The bridge between the PPDS method and the PLADMM is the Moreau decomposition [2]. Thus, the proposed method can also be seen as an extension of the PLADMM. The contribution of this paper is threefold:

1. A parallel primal-dual splitting framework is proposed to cover both the PDS method and the PLADMM. By imposing different weights onto each linear operator and applying a relaxed step, fast convergence speed is achieved. At each iteration of the derived algorithm, only proximity operators and forward linear operators are involved; thus, the proposed method possesses a highly parallel structure and can be accelerated with a highly parallelized hardware. By excluding the linear inverse operator, the proposed method is not partial to a particular data boundary condition.