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Asymmetric hesitant fuzzy sigmoid preference relations in the analytic hierarchy process

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ABSTRACT

This study considers the practical phenomena in the process of preference elicitation and proposes an asymmetric sigmoid numerical scale (ASNS) based on a generalized sigmoid function. It also offers proof of the scale's asymmetry, variability, consistency, and diminishing utility properties. Further, this study introduces the hesitant fuzzy preference format and defines the hesitant fuzzy continuous preference term. Based on this approach, the asymmetric hesitant fuzzy sigmoid preference relation (AHSPR) is developed and used in the analytic hierarchy process (AHP). The results show that the AHSPR is a general and optimal preference relation. Additionally, this study constructs a discrete fitting technology and an approximate translation method as the applied bases of the new scale and the preference relation. Following this, a model framework of the AHSPR in the AHP is provided. Finally, this study re-examines a well-known numerical example in order to demonstrate the application and advantages of the proposed numerical scale, the preference format, and the modeling framework.

1. Introduction

The analytic hierarchy process (AHP) is a well-known and effective decision-making technology. As such, it has had many applications and been the subject of numerous studies [3,29]. The AHP is built on human beings' intrinsic ability to construct their perceptions or ideas hierarchically. In this process, a decision maker (DM) first provides a pairwise comparison matrix (PCM) in accordance with one of the preference formats such as real, fuzzy, interval fuzzy, intuitionistic fuzzy, interval intuitionistic fuzzy, and hesitant fuzzy numbers. Then, the DM selects a numerical scale in order to quantify the PCM and obtain the numerical pairwise comparison matrix (NPCM). Finally, a priority vector is derived from the NPCM, following which the optimal alternative is presented. Thus, this process requires three main parts: the preference format, the numerical scale, and the prioritization approach.

With regard to the preference format, the real number is the conventional presentation. Saaty [23] used this as the basis for his proposal of the AHP and employed it in many fields of decision making over the last few decades [20–24]. Because of the complexity and uncertainty of real-life decision-making problems, various preference formats and the corresponding preference relations have been developed. These include the fuzzy preference format [16], the interval-valued fuzzy preference format [4], the interval-valued multiplicative preference format [14], the intuitionistic fuzzy preference format [37], the interval-valued intuitionistic fuzzy preference format [37], the intuitionistic multiplicative preference format [31], the

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Table 1
Meanings of the 1–9 scale.

The left part of the 1–9 scale		Preference terms	Preference terms	The right part of the 1–9 scale	
The values between 1/9 and 1, excluding 1, which refer to the non-preferred relations	1/9→	Absolutely non-preferred	Extremely preferred	←9	The values between 9 and 1, excluding 1, which refer to the preferred relations
	1/7→	Very strongly non-preferred	Very strongly preferred	←7	
	1/5→	Strongly non-preferred	Strongly preferred	←5	
	1/3→	Moderately non-preferred	Moderately preferred	←3	
Equal relation	1→	Equally non-preferred	Equally preferred	←1	Equal relation

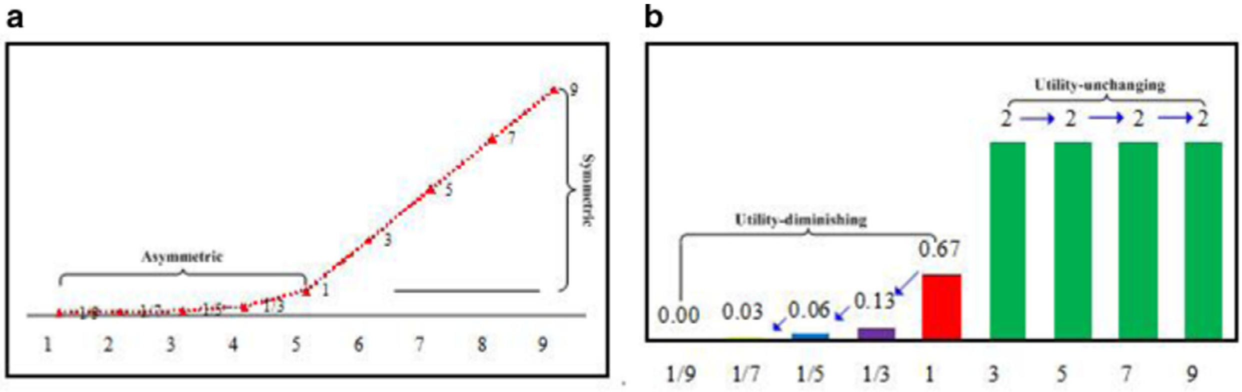

Fig. 1. Presentation of the 1–9 scale and its marginal utility.

hesitant fuzzy preference format [39], the interval-valued hesitant preference format [5], the hesitant fuzzy linguistic preference format [40], and the hesitant multiplicative preference format [38]. Of these, the hesitant fuzzy preference format is the more general numerical format and can be transformed into other fuzzy preference formats. Thus, we introduce this preference format as the basis of the proposed new preference relation.

Numerical scale is the second key aspect of preference relations, and mainly falls into two categories: the multiplicative 1–9 scale [19,23] and the reciprocal 0.1–0.9 scale [30,31]. Thus, based on these scales, preference relations can be classified into multiplicative preference relations and fuzzy preference relations. Other numerical scales have also been proposed, including the $2^{k/2}$ ($2^{0/2}$,$2^{2/2}$,$2^{4/2}$,$2^{6/2}$,$2^{8/2}$) numerical scale [23], the 1–5, the 1–15, the x^2 (1,9,25,49,81) and the $\sqrt{x}$ (1,$\sqrt{3}$,$\sqrt{5}$,$\sqrt{7}$,3) [11], the 9/9–9/1 [15], the Salo–Hämäläinen [25], the geometrical [12], the balanced [9], the verbal [10], and the individual numerical scales [7,8]. However, the 1–9 scale and the 0.1–0.9 scale are still the two most common ones. Furthermore, as Xia et al. [31] pointed out, Saaty's 1–9 scale is better for expressing preference relations than other numerical scales because information is usually distributed asymmetrically. Moreover, in our daily life, the law of diminishing marginal utility applies. Nonetheless, we find that these two properties cannot be represented comprehensively using the 1–9 scale (please see Section 2.1, Table 1, and Fig. 1 for more details). Additionally, two other practical properties, consistency and variability, should be introduced and considered in order to optimize the numerical scale. In this regard, consistency requires the numerical scale to be in a constant domain [0,1], which is also a basic condition for the fuzzy preference relations. For different DMs, distinct numerical scales may be provided in order to represent their various risk appetites. This demonstrates the numerical scale's variability. Thus, in order to further discuss these properties and combine them with other practical properties, we develop an asymmetric sigmoid numerical scale (ASNS).

The final key issue is the prioritization approach, which derives the priority vector from the NPCM. Many prioritization approaches have been developed. Among them, the eigenvalue [19,26], the logarithmic least squares [6,13], and the goal programming methods [36] are the commonly used approaches. However, in order to establish a new preference relation based on the ASNS and the hesitant fuzzy preference form, these approaches may be unsuitable for direct use. Thus, in this paper, we design a new prioritization approach in order to obtain the priority vector from the new NPCM and preference relations.

As aforementioned, the primary purpose of this study is to propose the ASNS with four desired properties and to introduce the hesitant fuzzy preference format, which is a general fuzzy presentation. Based on this approach, the asymmetric hesitant fuzzy sigmoid preference relation (AHSPR) is constructed in order to extend fuzzy preference relations with