Information Sciences 357 (2016) 61–87

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Proportional hesitant fuzzy linguistic term set for multiple criteria group decision making

Zhen-Song Chen [a,c], Kwai-Sang Chin [b], Yan-Lai Li [a,c,*], Yi Yang [a,c]

[a] School of Transportation and Logistics, Southwest Jiaotong University, Chengdu, Sichuan 610031, People's Republic of China
[b] Department of System Engineering and Engineering Management, City University of Hong Kong, 83 Tat Chee Avenue, Kowloon Tong, Hong Kong, People's Republic of China
[c] National United Engineering Laboratory of Integrated and Intelligent Transportation, Southwest Jiaotong University, Chengdu, Sichuan 610031, People's Republic of China

ARTICLE INFO

Article history:
Received 22 July 2015
Revised 22 March 2016
Accepted 2 April 2016
Available online 9 April 2016

Keywords:
Multiple criteria group decision making (MCGDM)
Proportional hesitant fuzzy linguistic term set (PHFLTS)
t-norm
t-conorm
Possibility degree formula

ABSTRACT

The theory of hesitant fuzzy linguistic term sets (HFLTSs) is a powerful technique used to describe hesitant situations, which are typically assessed by experts using several possible linguistic values or rich expressions instead of a single term. The union of HFLTSs with respect to each expert, that is, an extended HFLTS (EHFLTS), further facilitates the elicitation of linguistic assessments for addressing group decision-making problems because EHFLTSs can deal with generalized (either consecutive or non-consecutive) linguistic terms. In this study, we propose proportional HFLTSs (PHFLTSs), which include the proportional information of each generalized linguistic term. The mathematical form for a PHFLTS is consistent with that for a linguistic distribution assessment. However, the underlying meanings of the proportions associated with generalized linguistic terms are different. PHFLTSs can be viewed as a special method for performing linguistic distribution assessments. PHFLTSs are recognized as a useful extension of HFLTSs and a possibility distribution for HFLTSs under different assumptions. We define the basic operations with closed properties among PHFLTSs on the basis of t-norms and t-conorms. We then propose a probability theory-based outranking method for PHFLTSs by providing possibility degree formulas. We also study two fundamental aggregation operators for PHFLTSs, namely, the proportional hesitant fuzzy linguistic weighted averaging operator and the proportional hesitant fuzzy linguistic ordered weighted averaging operator. Several important properties of these aggregation operators are investigated. Finally, we use the proposed multiple criteria group decision-making model in practical applications.

1. Introduction

In 1965, Zadeh [57] presented the concept of linguistic variables, which carry values that comprise words or sentences rather than numbers, in natural or artificial language. Linguistic variables serve as an effective tool for representing qualitative aspects as linguistic values to facilitate decision-making [30,31]. To date, many linguistic models, particularly the

* Corresponding author at: School of Transportation and Logistics, Southwest Jiaotong University, Chengdu, Sichuan 610031, People's Republic of China. Tel.: +8613208183580.

E-mail addresses: czs@my.swjtu.edu.cn (Z.-S. Chen), mekschin@cityu.edu.hk (K.-S. Chin), yanlaili@home.swjtu.edu.cn (Y.-L. Li), yangshijiazu@my.swjtu.edu.cn (Y. Yang).

http://dx.doi.org/10.1016/j.ins.2016.04.006

three classical linguistic computational models (i.e., semantic model [4,42], symbolic model [5,16,54], and linguistic two-tuple model [18,19]), have been proposed to extend and improve the fuzzy linguistic approach from various perspectives. The two-tuple linguistic representation model proposed by Herrera and Martínez [18] has been utilized in a wide range of applications, especially in the fields of analytic hierarchy process [11–13,34], and multiple criteria group decision making (MCGDM) [9,19]. However, the Herrera and Martínez model is only suitable for linguistic variables with equidistant labels. Moreover, this model can only be used to deal with linguistic term sets that are uniformly and symmetrically distributed. In practice, linguistic labels may not be symmetrically distributed around a medium label and they need not meet the traditional requirement of linguistic labels being separated at "equal distances" between them. To address this drawback, Wang and Hao [45] proposed the proportional two-tuple linguistic representation model as an extension of the Herrera and Martínez model on the basis of the concept of symbolic proportion. As linguistic term sets are not usually uniformly and symmetrically distributed in real-life decision making situations, Herrera et al. [19] proposed the concept of unbalanced linguistic term sets and a fuzzy linguistic methodology to deal with it [20]. Furthermore, an integrated and extended model of the Herrera and Martínez model, the Wang and Hao model and the Herrera et al. model [19] were comprehensively investigated by Dong and Herrera-Viedma [9] and Dong et al. [6,9–12,14]. The aforementioned models have been employed in numerous areas and thus motivate the development of novel theoretical and practical techniques in information modeling and computing processes for linguistic expressions.

Most linguistic decision approaches handle linguistic terms with defined priori and thus prevent experts from utilizing rich expressions to articulate their preferences. The use of only one linguistic term for information modeling restricts experts from precisely expressing themselves [38]. These issues have been addressed with an increasing number of methods [28,45] that provide flexible and rich expressions comprising more than one linguistic term. However, Rodríguez et al. [37] argued that previous linguistic approaches are neither close to the cognitive models of human beings nor suitable expressions for linguistic preferences in group decision making (GDM). In view of these drawbacks, Rodríguez et al. [36] advocated the use of context-free grammar and further extended it [37] to generate comparative linguistic expressions. The expression domain generated by the extended context-free grammar consists of single-valued linguistic terms and comparative linguistic expressions. A transformation function was introduced to convert these expressions into hesitant fuzzy linguistic term sets (HFLTSs), which were developed on the basis of the fuzzy linguistic approach and the concept of hesitant fuzzy sets (HFSs) [37] because such expressions cannot be directly used for computing with words (CWW). HFLTSs assess linguistic variables using the comparative linguistic expressions generated by context-free grammar because experts doubt different linguistic terms or require complex linguistic terms to express their knowledge accurately. Therefore, HFLTSs increase the flexibility and capability of eliciting linguistic information [37,47,52,53]. HFLTSs have received increasing interest because of the immense quantity of their practical and potential applications, particularly in mathematics, computer science, economics and social sciences [15,21–25,27,29,40,48]. Wei et al. [47] identified two drawbacks in Rodríguez et al. [36]: the operations among HFLTSs do not exhibit a closed property, and the strict dominance relations between any two HFLTSs obtained with the comparison method on the basis of the envelopes of HFLTSs may be unreasonable if the two HFLTSs share common elements. To overcome these disadvantages, Wei et al. [47] defined the closed operations among HFLTSs and provided possibility degree formulas for comparing HFLTSs. They also proposed two useful aggregation operators for HFLTSs, namely, a hesitant fuzzy linguistic weighting averaging (HFLWA) operator and a hesitant fuzzy linguistic ordered weighting averaging (HFLOWA) operator, for decision making because min-upper and max-lower operators cannot deal with situations in which the important weights of criteria or experts are considered. However, the decision-making methods of Rodríguez et al. [36] and Wei et al. [47] cannot distinguish the preference order of alternatives in several situations. Wang et al. [49] further suggested a multiple criteria decision-making approach that combines HFLTSs with an outranking method involving systematic comparisons of the assessment values of alternatives for each criterion. The existing literature provides detailed information on the theoretical development and practical applications of HFLTSs, including the basic definitions and operations for HFLTSs [21]-[23,36,48], information fusion methods with HFLTSs [36,47], information measures of HFLTSs [22]-[24,25,51], and distinct decision-making methods [15,25,37]. In GDM problems, we may encounter situations in which individual priorities are completely unknown, particularly when individuals are allowed to provide their evaluation information anonymously. Thus, Wang [50] modeled complex linguistic assessments by proposing the concept of extended HFLTSs (EHFLTSs), which are characterized by a set of generalized (either consecutive or non-consecutive) linguistic terms, to perform group evaluation under GDM environments. According to the construction axiom of EHTLTSs, the union of HFLTSs leads to EHFLTSs. EHFLTSs exhibit more desirable mathematical properties and greater flexibility than HFLTS in managing the assessments of a group under uncertainties in anonymous situations. This difference can be explained by the fact that either a discrete subset or a consecutive subset of a given linguistic term set can be represented.

The concept of integrating proportional information into linguistic terms was initially elaborated by Wang and Hao in 2006 [45]. The Wang and Hao model assigns symbolic proportions to two successive ordinal terms in a linguistic term set. The model allows the case in which the grades of an individual in the answerscripts of a course are distributed as 20%A and 80%B to be expressed by a proportional two-tuple model, i.e., (0.2A, 0.8B). The symbolic proportions in a proportional two-tuple linguistic representation model are regarded equivalent to the weights of each generalized linguistic term. Inspired by the pioneering work of Wang and Hao, Zhang et al. [58] proposed the linguistic distribution assessment model, which is a natural generalization of the Wang and Hao model. The linguistic distribution assessment model assigns symbolic proportions to all linguistic terms more explicitly than it does when assigning symbolic proportions that greater than zero to generalized linguistic terms in a linguistic term set. In their original work, the proportional information is provided by an