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Hierarchical structures and uncertainty measures for intuitionistic fuzzy approximation space

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ABSTRACT

The hierarchical structures and uncertainty measures in granular computing are the two main aspects for investigating the structure and uncertainty of all types of approximation spaces. Although several hierarchical structures and uncertainty measures have been proposed to represent and analyze different granular structures, these structures and uncertainty measures are studied under crisp or fuzzy conditions. Hierarchical structures and uncertainty measures for the intuitionistic fuzzy (IF) approximation space are addressed in this paper. First, we propose the representation and operations of IF granular structures, as well as examine four hierarchical structures and a lattice structure of IF approximation space. Second, the natural extensions of fuzzy information granularity, fuzzy information entropy, fuzzy rough entropy, and fuzzy information Shannon entropy, namely, IF granularity, IF information entropy, IF rough entropy, and IF information Shannon entropy, are developed and adopted to characterize the uncertainty of IF granular structures in the IF approximation space. Third, we provide the multi-granulation IF approximation space and study its four hierarchical structures. Furthermore, we discuss the relationships between the presented hierarchical structures and multi-granulation IF rough sets in the multi-granulation IF approximation space. Fourth, we propose four types of IF uncertainty measures to depict the uncertainty of the multi-granulation IF knowledge base in the multi-granulation IF approximation space with respect to optimistic and pessimistic multi-granulation IF rough sets, which is more reasonable compared with previous work conducted in a crisp context.

1. Introduction

Granular computing (GrC), which was initiated by Zadeh [60], plays a fundamental role in intelligence information processing [6,11,19,21]. An information granule is the basic unit in GrC; it is a chunk of objects drawn together by equivalence, indistinguishability, similarity, or proximity of functionality [27,33,34]. The information granulation of all objects in the universe leads to a collection of granules called a granular structure. After information granulation, all objects in the universe of discourse are granulated into a family of disjoint or overlapping granules. GrC attempts to find some appropriate information granules that can effectively approximate a complex concept at a specific level of granulation or at many levels of multi-granulation. Since

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its proposal, GrC has become a fast-growing and hot research topic [15,18,31,35,39,40]. The information granulation of objects in a universe because of an equivalent relation is an equivalence granular structure, where two distinct information granules (equivalence classes) are disjointed [29,38]. However, two covering-relation-based information granules (tolerance classes) can intersect each other in an information granular structure induced by a covering relation [55]. All objects in a classic information system can generally be granulated easily into a binary granular structure induced by a binary relation (i.e., equivalence, similarity, dominance, covering, and neighborhood) [1,5,7,20,30,43,45,51,57,62,63,66]. All objects in a universe can also be granulated into a fuzzy binary granular structure induced by a fuzzy binary relation [12–14].

After all granular structures are constructed in an approximation space, the hierarchy structure of the approximation space is utilized to analyze the relationships among all granular structures in the approximation space [28]. For example, Yao [56] suggested the use of hierarchical granulations to study stratified rough set approximations, whereas Yang et al. [54] proposed three hierarchical structures of multi-granulation approximation space. She et al. [42] investigated the topological and lattice structures of multi-granulation approximation space. Another important concept in GrC is the uncertainty measure of a granular structure, which is a measure of uncertainty about the actual structure of the granular structure. Information granularity, information entropy, rough entropy, and information Shannon entropy are mainly expressions of uncertainty measures in various approximation spaces [10,23,24,32,49,57,64]. Information granularity generally represents the discernibility of an information granule in a granular structure. The smaller the information granularity of a granular structure, the stronger its discernibility becomes. Thus calculating the information granularity of a granular structure has always been an important issue based on the different views and targets [36,44,50]. The entropy of a granular structure, as defined by Shannon [41], is a useful mechanism for characterizing the information content of a granular structure. Shannon entropy is defined by the probability of a partition in the universe; therefore, it cannot be employed to depict the uncertainty of a granular structure induced by a general binary relation or a fuzzy binary relation. Shannon entropy is extended to different generalized granular structures called information entropy or information Shannon entropy, which are used to characterize the uncertainty of the generalized granular structures to overcome this drawback [37]. A special type of uncertainty measure called roughness exists in rough set theory. For a given granular structure, researchers use rough entropy to measure the roughness degree of the given granular structure [23,37].

Atanassov's IF sets were first formulated by Atanassov [2,3]. An IF set is a generalization of fuzzy sets introduced by Zadeh. The membership value, $\mu(x)$, of x in the universe of discourse U in fuzzy sets, is only a single real number, which is generally in [0,1], where the non-membership of x is taken as $1-\mu(x)$. However, the membership value $\mu(x)$ and the non-membership value $\gamma(x)$ for IF sets should be considered to describe any x in U such that the membership and non-membership sum is less than or equal to 1. Thus, an IF set is expressed with an ordered pair of real numbers $<\mu(x), \gamma(x)>$ where $0\le\mu(x)+\gamma(x)\le 1$, and $1-\mu(x)-\gamma(x)$ is called the degree of hesitancy. Although IF set theory has been successfully applied in decision analysis and pattern recognition [8,9,22,48,59,61], hierarchical structures and uncertainty measures of IF granular structures have scarcely been addressed recently [16].

Based on the research progress described above, many important results have been achieved in the domains of hierarchical structures and uncertainty measures of different approximation spaces. In some sense, without considering the applications, there are three basic problems in GrC: (1) information granulation: how to granulate all the objects in a universe into an information granular structure, (2) hierarchical structure: how to rank all the information granular structures in a granular space, and (3) information granularity: how to measure the granulation degree of an information granular structure. However, to the best of our knowledge, there did not exist a unified framework for these three developments, which is efficient for studying IF granular structures. This concept is the driving force for our research. This study intends to solve three key problems: (1) how to granulate all objects in a universe into an IF granular structure, (2) how to construct the hierarchical structure of IF approximation space, and (3) how to use uncertainty measures to depict the uncertainty of IF granular structures. We will address these three problems from single granulation and multi-granulation points of view.

The rest of this paper is organized as follows. Section 2 briefly introduces the preliminary issues considered in this study, such as the notions of fuzzy binary granular structures and their basic properties as well as IF sets and their operations, and provide the concept of an IF approximation space. Section 3 presents the IF granulation based on an IF binary relation and operators of IF binary granular structures and their four order relations, which constitute four hierarchical structures of the IF approximation space. Furthermore, a lattice structure of the IF approximation space is investigated. Section 4 discusses how to measure the uncertainty of an IF granular structure using four extended measures, namely IF granularity, IF information entropy, IF rough entropy, and IF information Shannon entropy of an IF granular structure as well as establishes the relationships among them. Section 5 presents four hierarchical structures of multi-granulation IF approximation space and discusses the relationships between the presented hierarchical structures of multi-granulation IF approximation space and multi-granulation IF rough sets in this multi-granulation IF approximation space. Section 6 develops four optimistic and pessimistic uncertainty measures for IF multi-granulation approximation space after carefully investigating the aforementioned fusion uncertainty measures for multi-granulation approximation space in a crisp context. Finally, Section 7 concludes this paper.

2. Preliminaries

This section recalls several basic concepts, such as fuzzy-binary granular structures and their basic properties, IF sets and their operations, and IF binary relations. The concept of an IF approximation space is then presented.

Definition 1 [12]**.** Given the universe of discourse $U=\{x_1,x_2,\cdots x_n\}$, a fuzzy binary relation $\tilde{R}$ can be denoted by the following matrix: