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Stability and stabilization of switched linear time-invariant systems with saddle points and switching delays

Liying Zhu*

College of Mathematics, Physics, and Information Engineering, Zhejiang Normal University, Zhejiang, Jinhua 321004, China

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ABSTRACT

This paper addresses global asymptotic stability (GAS) and stabilization issues of switched linear time-invariant (LTI) systems with saddle points and time-varying switching delays. Firstly, two criteria of GAS are proposed for such switched systems with all subsystems sharing a common unique saddle point under arbitrary *periodic/quasi-periodic switching paths (PSP/QSP)*. Secondly, by using our stability results we design *global asymptotic-stabilizing controls (GASC)*, *periodic/quasi-periodic stabilizing switching paths (PSSP/QSSP)*, and an algorithm of GASC and PSSP/QSSP for the switched systems. Finally, we present a numerical example of a switching multi-agent system to illustrate the effectiveness and practicality of our new results.

1. Introduction

Switched systems, an important kind of hybrid systems [3,7,13], have attracted much attention in control theory [3,4,11,26] and engineering communities [1] in the past 20 years. This may be due to that switched systems can be used widely to formulate many physical systems in nature and engineering [5,8]. Till now, there are many results on system analysis and synthesis obtained for switched systems, see for example [1,2,6,9–12,14,16,17,21–26] and the references therein. Meanwhile, several useful methods or techniques, such as the common Lyapunov function (CLF) method [4,11], the multiple-Lyapunov functions (MLF) method [3], the multiple-storage functions (MSF) method [26], the average dwell-time approach [24,27,28], and the linear matrix inequality (LMI) technique [8,9,15,24] are well developed for analyzing or designing switched systems. Among the results in the literature are lots of system analysis and design results on switched linear time-invariant (LTI) systems [4,10,12,15,18,25,30]. By means of the LMI technique, the average dwell-time approach, and other techniques, researchers have also obtained many results on system stability analysis and synthesis for switched systems with time delays or state-dependent delays [5,A1,7,9,A2,15,17,19,24].

However, it is well worth pointing out the following. Many stability and stabilization results of switched systems in the literature require classical stability analysis and design techniques or ideas based on suitable Lyapunov functions. Almost all of the stability results on switched systems in the literature are unsuitable for switched systems with all subsystems unstable. The general techniques or methods of system stability analysis and design for switched systems with all subsystems stable, such as CLF, MLF, MSF, LMI, and other classical methods, seem to be useless for analyzing stability of switched systems with all subsystems unstable. Indeed, when all the subsystems are unstable, the desired Lyapunov-type functions of subsystems cannot

* Tel.: +86 057982298188.
E-mail address: lyzhu@zjnu.edu.cn, lyzhu@zjnu.cn

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be obtained over their active state spaces. As the above techniques use Lyapunov functions or Lyapunov-type functions, they cannot be applied to switched systems with all subsystems unstable.

The motivation for this paper comes from the following. Switched systems with saddle points are used to formulate some practical engineering systems, such as electric circuits [1], mobile robot systems, power systems, etc. For instance, if some emergency or system failure occurs suddenly, every subsystem of second-order switching power systems may have a saddle point (a kind of improper equilibria). Due to the existence of a saddle point, such switching power systems may be unstable, and then voltage collapses of the systems may occur. From the viewpoint of system security, it is vital to avoid voltage collapses of switching power systems with saddle points. Moreover, voltage collapses caused by the existence of saddle points are natural stability issues of switched power systems with saddle points. However, without having workable Lyapunov functions for every subsystem, the existing methods or techniques based on Lyapunov functions cannot be used to analyze stability of switched systems with saddle points. In addition, switched systems with switching delays is also more complex than switched systems without delays. It is indeed more difficult and challenging to investigate switched systems with saddle points and switching delays. Although some local asymptotic stability (LAS) results are given for two-dimensional LTI switched systems with saddle points in [18], to the author's best knowledge, there are a few stability/stabilization results for switched LTI systems with saddle points and switching delays.

This paper investigates global asymptotic stability (GAS) and stabilization of switched LTI systems with saddle points and time-varying switching delays. The main contributions of this paper are several novel results on GAS and stabilization for switched systems with all the subsystems whose state matrices are all block matrices. By means of the concept of GAS and the criterion of direction of subsystems obtained in [18], we will first propose two sufficient conditions for GAS of switched LTI systems with saddle points under arbitrary *periodic/quasi-periodic switching paths (PSP/QSP)* with time-varying switching delays. Then, based on the new stability results obtained in this paper we will design *global asymptotic-stabilizing controls (GASC)*, *periodic/quasi-periodic stabilizing switching paths (PSSP/QSSP)*, and a corresponding algorithm for such switched control systems. Finally, the obtained stability and stabilization results will be applied to a switching multi-agent system (SMAS). The numerical simulations of the SMAS will then show the effectiveness and practicality of the new results.

Compared with existing stability and stabilization results on switched systems in the literature, the GAS and stabilization results presented in this paper have significant advantages and novelties as follows. The hypotheses in these results are relationships among the elements of subsystems state matrices, the switching dwell times, and the switching delays. Hence, the criteria only depend on the elements of the state matrices of subsystems and switching paths. Moreover, all the results presented in this paper are obtained without resort to any type of Lyapunov functions or Lyapunov-like functions. Most important, the conditions of the main results given in this paper are explicit, easily-tested, and practical.

The rest of the paper is organized as follows. Section 2 presents preliminaries including the system model, definitions, notation, and assumptions. In Section 3, several GAS and GASC-PSSP/QSSP results are proposed for switched LTI systems with saddle points and time-varying switching delays. An illustrative example of a switching multi-agent system is carried out in Section 4, which is followed by the conclusion in Section 5.

Notation: With $\mathbb{N}$ denoting the set of natural numbers and $\mathbb{N}_+$ denoting the set of positive integers, $[K]$ denotes the set $\{1, 2, \ldots, K\}$, where $K \in \mathbb{N}_+$. Also, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\|\cdot\|$ denotes the Euclidean norm in $\mathbb{R}^n$, and $\mathbb{R}^{m\times n}$ denotes the $m \times n$ matrix space of real matrices, with the superscript 'T' denoting matrix transposition. Finally, $<\cdot,\cdot>$ denotes the inner product, and $V_1 \oplus V_2$ denotes the direct sum of two vector spaces V_1 and V_2.

2. Preliminaries

Consider a switched LTI control system as follows

$$\dot{x} = A_{\sigma(t)}x + B_{\sigma(t)}u_{\sigma(t)}, x(t_0) = x(0), \quad \text{for} \quad t \geqslant t_0. \tag{1}$$

Here $x = [(x^1)^T, \ldots, (x^n)^T]^T \in \mathbb{R}^{2n}$ is the state of the system, where $x^h = [x_1^h, x_2^h]^T \in \mathbb{R}^2$ for $h \in [n]$. The map $\sigma : [t_0, +\infty) \to [N]$ is a piecewise right-continuous step function called a *switching path* or *switching rule*, where N is an integer larger than 1. The value $\sigma(t) = i$ means that subsystem i is active. The positive time delay τ_i caused by the switching path σ is called the *switching delay* of subsystem i, which may be time-varying. The state matrix $A_i = \text{diag}\{A_1^i, \ldots, A_n^i\}$ and the input matrix $B_i = \text{diag}\{B_1^i, \ldots, B_n^i\}$ of subsystem i are $2n \times 2n$ matrix and $2n \times n$ matrix, respectively, where $A_h^i = (a_{wvh}^i)_{2\times 2} \in \mathbb{R}^{2\times 2}$, $B_h^i = [b_{h1}^i b_{h2}^i]^T \in \mathbb{R}^{2\times 1}$, for $w, v \in \{1, 2\}$, $i \in [N]$, and $h \in [n]$. The $1 \times 2n$ vector u_i is the input of subsystem i. Particularly, when $u_i \equiv 0$ system (1) becomes a switched LTI system as follows:

$$\dot{x} = A_{\sigma(t)}x, x(t_0) = x(0), \quad \text{for} \quad t \geqslant t_0. \tag{2}$$

Corresponding to the parts A_h^i of all the matrices A_i of its subsystems, for system (2) there are nN second-order systems as follows

$$\dot{x}^h = A_h^i x^h, x^h(t_0) = x^h(0), \quad \text{for} \quad t \geqslant t_0, i \in [N], \quad \text{and} \quad h \in [n], \tag{3}$$

where $x^h = [x_1^h, x_2^h]^T \in \mathbb{R}^2$. Note that systems (3) are in fact n second-order LTI switched systems scheduled by the same switching path σ of systems (1) and (2).

When the parts A_h^i of the matrices A_i are all block-triangular for $h \in [n]$, we let

$$[N]_h^u = \{i \in [N] : a_{11h}^i a_{22h}^i < 0, \ a_{12h}^i \neq 0, \ a_{21h}^i = 0\} \tag{4}$$