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Recursively spreadable and reductible measures of specificity

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ABSTRACT

The expansible and recursively spreadable measures of specificity are introduced to improve the performance of computing measures of specificity of fuzzy sets when the universe of candidates is increased. The contractible measures of specificity and recursively reductible measures of specificity again improves the performance to compute the measure of specificity of a fuzzy set when the universe of candidates is decreased from the previous one. These kind of measures of specificity allows to compute very efficiently measures of specificity of fuzzy sets when a universe of discourse is variable.

1. Introduction

Measures of specificity [5,6,9–13] are useful measures of information for fuzzy sets that allow to know how useful they are to choose an element from the universe. They have also been defined for interval-valued fuzzy sets, intuitionistic fuzzy sets and type-2 fuzzy sets [7,14].

In real life, a universe of candidates can grow up or can be decreased, specially when it is needed to choose a subset of it. Yager's Expansible measures of specificity [15] do not change when a new candidate has been added to the universe of discourse with a membership degree zero. We now define the contractible measures of specificity as a dual concept, but now rejecting a candidate with membership degree zero from the universe.

On other side, Montero et al. [2–4] defined the recursive aggregation operators reducing the computational cost of aggregating $n+1$ values when the aggregation of the first n values is already known introducing the concepts of high performance operators when the universe is growing, using the previous calculations.

In [8] these ideas are joined to define recursively spreadable and recursively reductible measures of specificity which can be useful when we are computing the specificity measure of a fuzzy set and the universe is increased or decreased using a constant bounded number of operations and without having to resort of all the membership degrees of the expanded or reduced fuzzy set and so, allowing to compute high performance measures of specificity when the universe of candidates is changed.

The main utility of this work is to improve the performance to compute measures of specificity of fuzzy sets or possibility distributions when the universe of discourse is changed. Measures of specificity can be especially useful measure how easy is the selection of a candidate. When the set of candidates is big and it is still open to add or withdraw candidates, new methods to efficient computations the measures must be considered using only the previous computations and the inserted or deleted data.

This paper can be situated in a more general framework of measures of information and aggregation rules [1] that can reach good computation performance when the universe is increased or reduced. As it is usual to happen in transactional relational

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databases and analytic data warehouses where data is continuously inserted, deleted or updated and some information values must be maintained having profit of the already computed values and without recomputing a possibly big set of old values. Another example is Apache Hadoop big data environments and the map reduce programming paradigm, where the data and its computations are distributed in many parts in parallel and the reduced into one value from the previous computations. In those environments is typical to have ever increasing data, and new methods for high performance computations that should be maintained without accessing a huge set of older data are taking increasing importance.

2. Preliminaries

Definition 1. A fuzzy set μ on $X = \{e_1, \ldots, e_n\}$ is normal if there exists an element e_i in X such that $\mu(e_i) = 1$.

Definition 2. A fuzzy set μ on $X = \{e_1, \ldots, e_n\}$ is a singleton if there exists an element e_i in X such that $\mu(e_i) = 1$ and $\mu(e_j) = 0$ for all elements e_j in X with $i \neq j$.

Definition 3. [11] Let $X = \{e_1, \ldots, e_n\}$ be a finite universe and let $[0, 1]^X$ be the class of fuzzy sets on X. A measure of specificity Sp is a mapping Sp: $[0, 1]^X \to [0, 1]$ such that:

- $Sp(\mu) = 1$ if and only if μ is a singleton.
- $Sp(\emptyset) = 0$.
- If μ and ν are normal fuzzy sets and $\mu \subseteq \nu$, then $Sp(\mu) \geq Sp(\nu)$.

Definition 4. [11] The linear measure of specificity of a fuzzy set A on a finite space $X = \{e_1, \ldots, e_n\}$ is:
$Sp_{\vec{w}}(A) = a_1 - \sum_{j=2}^{n} w_j a_j$
where a_j is the jth greatest membership degree of A and $\{w_j\}$ is a set of weights verifying:

- $w_j \in [0, 1]$
- $\Sigma_{j=2}^{n} w_j = 1$

Definition 5. A linear measure of specificity is uniformly weighted if all its weights are equal.

Lemma 1. *Let Sp be a uniformly weighted measure of specificity on a finite space $X = \{e_1, \ldots, e_n\}$. Then $\{w_j\} = 1/(n-1)$ for all $j : 2, \ldots n$. So the uniformly weighted linear measure of specificity is:*

$$Sp(A) = a_1 - \sum_{i=2}^{n} \frac{a_i}{n-1}.$$

Definition 6. [15] The fractional measure of specificity is:
$Sp(A) = \frac{a_1^2}{\sum_{j=1}^{n} a_j}$
where a_j is the jth greatest membership degree of A.

Definition 7. [11] The product measure of specificity is:
$Sp(A) = a_1 \prod_{j=2}^{n} (ka_j + (1 - a_j))$
where k is in [0,1].

Definition 8. [15] Let A be an fuzzy set on $X = \{e_1, \ldots, e_n\}$ and let A' be an extended fuzzy set of A on $X' = \{e_1, \ldots, e_n, e_{n+1}\}$ with $A(e_i) = A'(e_i)$ for all e_i in X. A measure of specificity Sp is expansible if it satisfies the following condition:
If $A'(e_{n+1}) = 0$ then $Sp(A) = Sp(A')$.

3. Recursively spreadable measures of specificity

The recursively spreadable measure of specificity extend the Yager's concept of expansible measures of specificity but now allowing to easily compute the measure when a new candidate with any membership degree is added into the universe.

Definition 9. Let A be an fuzzy set on $X = \{e_1, \ldots, e_n\}$ and let A' be an extended fuzzy set of A on $X' = \{e_1, \ldots, e_n, e_{n+1}\}$ with $A'(e_i) = A(e_i)$ for all e_i in X and with $A'(e_{n+1}) = b$. A measure of specificity Sp is recursively spreadable if it satisfies the following axioms:

1. There exists a mapping ϕ_A: $[0, 1]^2 \to [0, 1]$ such that:

$$Sp(A') = \phi_A(Sp(A), b)$$

2. The computation of ϕ_A has a constant computational complexity $\mathcal{O}(1)$.

The mapping ϕ_A in Definition 9 is called the spread mapping of Sp.

Note that a constant computational complexity forces to have a finite bounded number of operations not depending on n, where n is the cardinality of the universe X.