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Complex-valued Bayesian parameter estimation via Markov chain Monte Carlo

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ABSTRACT

The study of parameter estimation of a specified model has a long history. In statistics, Bayesian analysis via Markov chain Monte Carlo (MCMC) sampling is an efficient way for parameter estimation. However, the existing MCMC sampling is only performed in the real parameter space. In some situation, complex-valued parametric modeling is more preferable as complex representation brings economies and insights that would not be achieved by real-valued representation. Therefore, to estimate complex-valued parameters, it is more convenient and elegant to perform the MCMC sampling in the complex parameter space. In this paper, firstly, based on the assumption that the observation signal is proper, two complex MCMC algorithms using the Metropolis–Hastings sampling and the differential evolution are proposed, in which the probability density functions (pdfs) in Bayesian estimation are characterized by the usual Hermitian covariance matrices. Secondly, to improve the performance for the case that the observation signal is improper, two augmented complex MCMC algorithms are developed, where the pdfs are computed by the augmented complex statistics. Both theoretical studies and numerical simulations are presented to show the effectiveness of the proposed algorithms in complex-valued parameter estimation.

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1. Introduction

Parametric models provide powerful tools for fitting experimental observations. Consequently, the estimation of parameters from noisy observations of a specified parametric model is significant. Currently, many approaches have been proposed [4,20,26]. In statistics, we usually treat the observations with disturbances and measurement errors as realizations, and then estimate the parameters using statistical inference. Bayesian analysis is such a kind of method [4,12,26]. Given a certain amount of observations, Bayesian approach gives the posterior density of the parameter of interest by combining the data likelihood with a prior distribution using the Bayes' rule. By taking account of the uncertainties related to models and parameters, a more accurate estimate may be achieved [12,24]. Besides, owing to the incorporation of the prior information of the parameters, Bayesian approach can reduce the sensitivity to the finite data length.

Although Bayesian approach has the aforementioned attractive properties, it meets some difficulties in evaluating the posterior densities, marginal of them, and the associated functions such as posterior means and covariance [24], as complicated integrals over arbitrary probability distributions are required. These integrals are in general analytically intractable, which limit the application of Bayesian approach to a wide range of complicated models.

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To tackle with this problem, many approaches have been proposed. The so-called 'Markov chain Monte Carlo (MCMC)' sampling, a kind of simulation method, is effective in handling these intractable integrals [13,25,29,32]. It can facilitate the implementation of Bayesian analysis of complex data sets containing missing observations and multidimensional outcomes. The MCMC approaches numerically compute the required density by accessing to random sequences (Markov chains) as realizations of the parameters, which converge to an invariant density that equals to the desired posterior density (target). Then, the desired summaries of the posterior distribution can be computed from the sample sequences correspondingly.

On the other hand, a variety of practical systems, such as those in communication, electromagnetic, oceanography and optics, are more preferable to be described by complex-valued parametric models [1,2,14,19,30]. Traditionally, to estimate the complex-valued parameters, we usually formulate the complex-valued signals as bivariate and real-valued signals, and then perform the MCMC sampling directly in real parameter space. This is natural as any linear complex-valued variable has a bivariate real-valued equivalent, though not vice versa. But it is inexact. Complex-valued representation in many cases is more elegant than real-valued representation. For example, it is more simple and efficient to present widely-linear or nonlinear transformations. Besides, it also brings economies and insights into the underlying physics that would not be achieved by real-valued representation [1,2,30]. So, it is more preferable to perform the MCMC sampling in complex space as to the complex-valued parameter estimation.

Considering this, in this paper, the estimation of complex-valued parameters from noisy observations of a complex-valued parametric model is studied using the framework of Bayesian analysis via complex MCMC sampling. To resolve the computational difficulties in Bayesian estimation, some complex MCMC methods using the Metropolis–Hastings (MH) sampling [15,22,23] and the differential evolution (DE) [5,6,28,31] are individually proposed to generate a series of samples that yield an invariant distribution of the desired posterior. After that, we can obtain the Bayesian point estimate using the posterior mean from the samples. Based on the assumption that the observation signal is proper (a complex random variable is uncorrelated with its complex conjugate), we directly extend the Bayesian estimation via MCMC in real parameter space to complex parameter space, in which the probability density functions (pdfs) are characterized by the usual Hermitian covariance matrices. The convergence analysis is then performed by resorting to its equivalent real-valued representation. In addition, considering that some practical system output signals are complex improper (a complex random variable is dependent on its complex conjugate), some augmented complex MCMC algorithms are further proposed, in which the pdfs in Bayesian estimation are computed by the augmented complex statistics. Then, a series of simulations are performed to show the effectiveness of the proposed algorithms in complex-valued parameter estimation.

The rest of the paper is organized as follows. In Section 2, some preliminaries of complex random variable or vector are introduced. In Section 3, the Bayesian approach for complex-valued parameter estimation is formulated. In Section 4, two complex-valued MCMC algorithms based on the MH sampling and the DE are proposed to implement the Bayesian estimation, and their convergence analyses are given in the same section. In Section 5, some augmented complex MCMC algorithms are further developed to improve the performance when the observation signal is improper. In Section 6, some simulation examples are presented to show the effectiveness of the proposed algorithms, and some conclusions are drawn in Section 7.

2. Preliminaries

To begin with, some preliminaries related to complex random variable or vector are introduced [1,30].

Consider a complex scalar, $x = x_r + jx_i$, where x_r and x_i are the real and the imaginary part of x respectively, and $j = \sqrt{-1}$ is the imaginary unit. For a differentiable function $f(x)$, the generalized complex derivative and the conjugate generalized complex derivative are defined as [36]:

$$\frac{\partial f(x)}{\partial x} \triangleq \frac{1}{2}\left(\frac{\partial f(x)}{\partial x_r} - j\frac{\partial f(x)}{\partial x_i}\right), \quad \frac{\partial f(x)}{\partial x^*} \triangleq \frac{1}{2}\left(\frac{\partial f(x)}{\partial x_r} + j\frac{\partial f(x)}{\partial x_i}\right). \tag{1}$$

Consider a D-dimensional complex random vector $\boldsymbol{x} = \boldsymbol{x}_r + j\boldsymbol{x}_i$, where $\boldsymbol{x}_r$ and $\boldsymbol{x}_i$ are a pair of D-dimensional real random vectors. We can construct two vectors closely related to $\boldsymbol{x}$. The first one is the *real composite* random vector, $\boldsymbol{\theta} = [\boldsymbol{x}_r^{\mathrm{T}}, \boldsymbol{x}_i^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{2D}$, which is obtained by stacking the real part $\boldsymbol{x}_r$ into the imaginary part $\boldsymbol{x}_i$. The second one is the *augmented complex* random vector, $\underline{\boldsymbol{x}} = \left[\boldsymbol{x}^{\mathrm{T}}, \boldsymbol{x}^{\mathrm{H}}\right]^{\mathrm{T}} \in \mathbb{C}_*^{2D}$, which is obtained by stacking $\boldsymbol{x}$ on top of its complex conjugate $\boldsymbol{x}^*$, where $\boldsymbol{x}^{\mathrm{H}} = (\boldsymbol{x}^{\mathrm{T}})^*$ denotes the Hermitian (conjugate) transpose.

The probability distribution (density) of a D-dimensional complex random vector ($\boldsymbol{x}$) is interpreted as the $2D$-dimensional joint distribution (density) of its real part ($\boldsymbol{x}_r$) and imaginary part ($\boldsymbol{x}_i$). That is, if $p(\boldsymbol{x})$ exists, we have

$$p(\boldsymbol{x}) = p(\boldsymbol{x}_r + j\boldsymbol{x}_i) \triangleq p(\boldsymbol{x}_r, \boldsymbol{x}_i). \tag{2}$$

Let $p(\boldsymbol{x})$ be the pdf of a complex random vector $\boldsymbol{x}$. According to (2), the expectation of $\boldsymbol{x}$ can be computed by

$$\begin{aligned} E[\boldsymbol{x}] &= E[\boldsymbol{x}_r] + jE[\boldsymbol{x}_i] \\ &\triangleq \int_{\mathbb{R}^{2D}} \boldsymbol{x}_r p(\boldsymbol{x}_r, \boldsymbol{x}_i) d\boldsymbol{x}_r d\boldsymbol{x}_i + j\int_{\mathbb{R}^{2D}} \boldsymbol{x}_i p(\boldsymbol{x}_r, \boldsymbol{x}_i) d\boldsymbol{x}_r d\boldsymbol{x}_i. \end{aligned} \tag{3}$$

For a complex random vector $\boldsymbol{x} = \boldsymbol{x}_r + j\boldsymbol{x}_i$, its complex *augmented covariance matrix* is computed by

$$\underline{\mathbf{R}}_{\boldsymbol{xx}} = E[(\underline{\boldsymbol{x}} - E[\underline{\boldsymbol{x}}])(\underline{\boldsymbol{x}} - E[\underline{\boldsymbol{x}}])^{\mathrm{H}}] = \begin{bmatrix} \mathbf{R}_{\boldsymbol{xx}} & \widetilde{\mathbf{R}}_{\boldsymbol{xx}} \\ \widetilde{\mathbf{R}}_{\boldsymbol{xx}}^* & \mathbf{R}_{\boldsymbol{xx}}^* \end{bmatrix}, \tag{4}$$

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