Information Sciences 324 (2015) 44–59

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

On multicriteria decision making under conditions of uncertainty

J.G. Pereira Jr.[a], P.Ya. Ekel[b,c,*], R.M. Palhares[d], R.O. Parreiras[b,c]

[a] Graduate Program in Electrical Engineering, Federal University of Minas Gerais, Av. Antônio Carlos, 6627, 31270-901 Belo Horizonte, MG, Brazil
[b] Graduate Program in Electrical Engineering, Pontifical Catholic University of Minas Gerais, Av. Dom José Gaspar, 500, 30535-630 Belo Horizonte, MG, Brazil
[c] ASOTECH, Av. Prudente de Morais, 290/507, 30380-002 Belo Horizonte, MG, Brazil
[d] Department of Electronics Engineering, Federal University of Minas Gerais, Av. Antônio Carlos, 6627, 31270-901 Belo Horizonte, MG, Brazil

ARTICLE INFO

Article history:
Received 28 October 2014
Revised 23 May 2015
Accepted 11 June 2015
Available online 2 July 2015

Keywords:
Multicriteria decision making
Information uncertainty
Payoff matrix
Choice criterion
Fuzzy decision making

ABSTRACT

This paper deals with multicriteria decision making problems under conditions of uncertainty. The main its contribution is the consideration of choice criteria of the classic approach to handle information uncertainty in monocriteria decision making as objective functions within the framework of multiobjective models, whose analysis generates harmonious solutions. Such consideration of choice criteria is of a fundamental character and allows one to modify the generalization, originally proposed by Ekel, Martini, and Palhares (2008), of the classic approach to handle information uncertainty for solving multicriteria problems. The modification permits one to overcome limitations of the indicated generalization, which can lead to contradictory decisions. Details of using the modification in a general scheme of multicriteria decision making under uncertainty are presented. The general scheme is focused on the use of available quantitative information to the highest degree to reduce decision uncertainty regions. However, if the problem solving capacity concerning quantitative information processing does not allow one to obtain unique solutions, the general scheme assumes the use of qualitative information based on knowledge, experience, and intuition of experts involved in the decision making process. Examples are presented to illustrate the modification of generalizing the classic approach as well as the general scheme of multicriteria decision making under information uncertainty.

1. Introduction

In the process of posing and solving a wide range of problems of an optimization character related to the design, planning, operation, and control of complex systems one inevitably encounters diverse types of uncertainty [1,11,19,38,45]. The authors of [8,38] distinguish two classes of uncertainties in decision making: internal uncertainties (related to decision maker (DM) values and judgments) and external uncertainties (defined by environmental conditions lying beyond the control of the DM). Although this paper deals with both kinds of uncertainties, the main its purpose is to handle the external uncertainties in problems of a multicriteria nature. Taking this into account, we can list two categories of situations, which require the use of a multicriteria approach [12,33]:

* Corresponding author at: Graduate Program in Electrical Engineering, Pontifical Catholic University of Minas Gerais, Av. Dom José Gaspar, 500, 30535-630 Belo Horizonte, MG, Brazil. Tel.: +55 31 3319 4305; fax: +55 31 3319 4225.
E-mail address: ekel@pucminas.br (P.Ya. Ekel).

http://dx.doi.org/10.1016/j.ins.2015.06.013
0020-0255/

- problems in which solution consequences cannot be estimated on the basis of using a single objective. These problems are related to the analysis of models which include economic as well as physical indices (when alternatives cannot be reduced to a comparable form) or indices whose cost estimations are hampered (for example, many power engineering problems are to be considered on the basis of technological, economical, ecological, and social nature criteria);
- problems that can be solved on the basis of using a single objective or even several objectives. However if the uncertainty of information does not permit one to derive unique solutions, then these problems can be rewritten as multicriteria problems by including additional criteria, which may have a qualitative nature, being based on the knowledge, experience, and intuition of involved experts (for example, "complexity of maintenance", "attractiveness of investments", etc.). This can serve as a useful means to contract the corresponding decision uncertainty regions.

Therefore, two classes of multicriteria models can be constructed [12,33]: $\langle X,M\rangle$, referring to multiobjective models, and $\langle X,R\rangle$, referring to multiattribute models.

When analyzing $\langle X,M\rangle$ models, a vector of objective functions $F(X) = \{F_1(X),\dots,F_q(X)\}$ is considered and the problem consists in simultaneous optimizing all objective functions, i.e.,

$$F_p(X) \to \underset{X\in L}{\text{extr}}, \quad p = 1,\dots,q \tag{1}$$

where L is a feasible region in $\mathbf{R}^n$.

An important step in analyzing the problem (1) is the determination of a set of Pareto solutions $\Omega \subseteq L$ [31]. This step is useful. However, it does not permit one to obtain unique solutions. As consequence, it is necessary to choose a particular Pareto solution taking into account the information provided by the DM. Three approaches to using this information are classified in [6,17,33]: *a priori*, *a posteriori*, and adaptive.

When analyzing multiobjective problems, it is necessary to develop answers to some specific questions. Among these questions, it is important to raise the normalization of objective functions, consideration of the importance or priority of each objective function, and the selection of a principle of optimality. The answers to these questions and, subsequently, the development of multiobjective methods are carried out in several directions ([9,30,36], for instance). But, without going into further discussions of that subject, it should be stressed that an important issue in the multiobjective analysis is the quality of obtained solutions. The quality is considered high if levels of satisfying objective functions are equal or close to each other, when all objectives have the same importance (refer to the concept of harmonious solutions [12,13]). This concept can be extended to handle objective functions with different importance levels [33]. At this point, it is important to stress the validity and advisability of the direction related to the principle of guaranteed result [12,33], which can be implemented by applying the Bellman–Zadeh approach to decision making in a fuzzy environment [2,34,44]. Its use permits one to realize an effective (from the computational standpoint) as well as rigorous (from the standpoint of obtaining solutions $X^0 \in \Omega \subseteq L$) method of analyzing multiobjective models. Its application allows one to preserve a natural measure of uncertainty in decision making and to take into account indices, criteria, and constraints of qualitative character.

Diverse aspects of utilizing the Bellman–Zadeh approach in the analysis of $\langle X,M\rangle$ models are discussed in [12,17,33]. The use of the results of [12,17,33] for solving diverse classes of power engineering problems is considered in [4,5,14].

On the other hand, the author of [38] indicates that many multicriteria models are based essentially on deterministic evaluations of the consequences of each action in terms of each criterion, possibly subjecting the final results and recommendations to a degree of sensitivity analysis. The use of such an approach may be justified when the primary source of complexity in decision making is related to the multicriteria nature of the problem rather than to the uncertain nature of individual consequences. However, in the situations when risks and uncertainties are as critical as issues of conflicting goals (for examples, [29,37,38]), more formal uncertainty modeling is needed [38]. This important consideration is consistent with the view of the authors of [33], where two principal ways of solving problems under conditions of uncertainty are distinguished. In applying the first way, one obtains (at least, theoretically) an exact solution for fixed values of the uncertain parameters, and then estimates its stability for variations of these parameters (for example, by performing multivariant computations). The second way presupposes the tracking of the effect of the uncertainty at all stages along the path toward the final decision. This way is more complicated than the first one, but is also more fruitful and highly promising. It allows one to increase the adequacy of built models and, as consequence, the credibility and factual efficiency of decisions based on their analysis. Taking this into account, it should be noted that the results of [15,16] may serve as a methodological tool for implementing the second way in the consideration of uncertainty in multicriteria decision making.

The results of [15,16] are associated with the generalization of the classic approach [3,28,35] to deal with the information uncertainty, based on building and analyzing payoff matrices reflecting effects which can be obtained for different combinations of solution alternatives and the so-called states of nature or scenarios, in monocriteria decision making to multicriteria problems.

The results of [15,16] are based on combining two branches of mathematics of uncertainty: elements of game theory and fuzzy set theory and do not fit the general approaches to deal with external uncertainties in the multicriteria analysis, classified and analyzed in [8,38] and discussed in more recent works [10,20]. These results, combined with the analysis of $\langle X,M\rangle$ models and $\langle X,R\rangle$ models (related to individual or group decision making based on fuzzy preference modeling [18,21,22,24,32,33], for instance; considering that $\langle X,R\rangle$ models are used for analyzing multiattribute problems, other approaches [26,27,42], for example, may be applied as well) served for developing a general scheme of multicriteria decision