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Dynamic estimation of the discernment frame in belief function theory: Application to object detection

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ABSTRACT

Classification or estimation problems deal with decisions among different hypotheses. In several applications, the set of hypotheses may evolve with time and/or with context. Then, this work focuses on the problem of dynamic estimation and update of the discernment frame in the framework of belief function theory.

Belief function theory is widely used in decision systems because of its ability to model both the imprecision and the uncertainty. Now, the problem of the discernment frame estimation is even more critical as the set of handled hypotheses is the power set of the discernment frame.

This study describes a solution to update and adjust the discernment frame in a sequential way as new sources provide new pieces of information. Besides incompleteness, it is assumed that the current discernment frame may contain duplicated or spurious hypotheses. We thus propose new update mechanisms and we show on a practical application, namely video surveillance, how these mechanisms may be applied.

1. Introduction

Although being the first and the most used theory for representing uncertainty, the probability theory may not be flexible enough when some lack of knowledge occurs. Indeed, it mainly represents the uncertainty of the information, whereas the information may be partial and/or imprecise. To model imprecision and partial ignorance in addition to uncertainty, the evidence theory was first introduced by Dempster [2] and then expanded by Shafer [19]. It generalizes the probability theory by allowing us to handle not only singleton hypotheses (as in the case of probability) but also compound hypotheses (i.e. disjunctive sets of hypotheses).

The set of the considered hypotheses is referred the discernment frame and denoted Ω. Its elements are the singleton hypotheses so that they are mutually disjoint. Under the assumption of a closed world, Ω is also assumed exhaustive, that is to say Ω includes all possible hypotheses for the variable of interest. Smets [20] proposed rather to consider an open world. Then, the belief in a missing hypothesis will be represented by allocating a non-null value to the mass function on the empty set $\emptyset$. Now, the interpretation of a non-null *mass* on the empty set is ambiguous since it can be derived either from a conflict between the sources or from a modeling problem (e.g., some missing hypotheses). To overcome this ambiguity, some

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authors proposed [8] to add a specific hypothesis (different from $\emptyset$) representing explicitly any missing hypothesis, and they called the associated discernment frame the 'extended open world'. Now, with such approach, we cannot distinguish between several 'missing' hypotheses. Then an alternative is to estimate the actual set of hypotheses to derive the right discernment frame.

Several authors worked on the automatic estimation of the discernment frame. A typical case of application is when the sensors are homogeneous [10,12] and complementary (i.e. they have different discriminative capacities). In the work [10], each source is defined on a discernment frame that is a subset of the common frame including all hypotheses considered by any source. Two deconditioning methods allow redefining the sources on the common frame. The first one is based on the Principle of Minimum Information introduced by Smets [20]. The second deconditioning method uses additional information about compatibility relationships between the hypotheses discerned by a source and the hypotheses that cannot be discerned. In the study [12], the discernment frame is derived in an unsupervised way from the intersections between the hypotheses respectively distinguished by individual sources.

The study [18] deals with the case of potentially heterogeneous sources (i.e. sources that may be related to different variables of interest). The common discernment frame should then be a Cartesian product between the discernment frames associated to each subset of sources homogenous in itself and heterogeneous with the others. According to Schubert [18], the axes of the product discernment frame should be exclusive sets and mutually disjoint. Given a product discernment frame, a set of tighter discernment frames, called abridgments, may be constructed, such that an abridgment is obtained by reducing at least one axis of the Cartesian product to one of its non empty subsets. A measure, called the Frame Appropriateness, is defined [18] to evaluate the quality of each abridgment. This method was illustrated on a toy example where three sources provide information regarding the color (for the first source), the speed (for the second source) and again the color (for the third source) of a car.

Conversely to the previously mentioned works, we assume here that new pieces of information occur (e.g., new sources are considered sequentially in the fusion process), which may induce not only an update of the discernment frame but also of the basic belief assignments. Let us give two examples of applications. Firstly, considering the classification problems, as data is acquired, the information becomes more complete and more accurate, so that new classes appear. Also, some non obvious links between classes are discovered so that some classes may be fused in a single class (for instance, in the *XIX*th century, the naturalist Huxley demonstrated the deep kinship between reptiles and birds, and he gathered them in a group called the Sauropsids). Secondly, considering the object detection problem, as data is acquired, some objects assumed as different are recognized as different parts of a same object partially hidden, and some new objects are detected.

In this paper, that extends our preliminary works [16,17], we consider video surveillance application as an applicative illustration of the proposed methodology to revise the discernment frame in the case of belief function framework. Our approach is more general than previously mentioned works in the sense that it handles three kinds of modification: the addition of new elements of Ω (i.e. new hypotheses), the removal of some Ω elements, the fusion of some existing elements.

The paper is organized as follows. Section 2 briefly recalls the operators of the belief functions we needed, and introduces the used notations. Section 3 specifies the problem. Section 4 presents the proposed approach and the derived methodology. Section 5 presents some results obtained on simulated data and on an actual video surveillance sequence. Section 6 summarizes the key points of this study and enumerates some perspectives of our work.

2. Background

In this section, we present the tools used in the following parts of this study. The definitions we present, as well as the interpretations of belief functions are those developed by Smets in his transferable belief model [20].

Let Ω denote the discernment frame, i.e. the set of mutually disjoint hypotheses possible for the variable of interest, and 2^{Ω} the set of Ω subsets. Three basic belief functions, related by one to one relationships, are defined from 2^{Ω} to $[0,1]$: the mass function m, the belief function *bel* and the plausibility function *pl*. For any subset A of Ω, they respectively represent the belief in A which cannot be assigned to any of its subsets, the minimal belief committed to A and the maximal belief that can be transferred to A.

The basic belief assignment (bba) m is such that the sum on subsets of Ω is equal to 1. The hypotheses for which $m(A) > 0$ are the focal elements. Let us introduce some popular bbas defined in terms of focal element(s). When Ω is the only focal element, the bba models complete ignorance and is called the 'void' bba. A bba with only two focal elements including Ω is called a Simple Support Function: When A is the supported hypothesis, it is noted A^w and $m(A) = 1 - w, m(\Omega) = w$ with $w \in]0,1]$. Under the open world assumption, $\emptyset$ may be a focal element and $m(\emptyset)$ is usually called Dempster's conflict.

Pignistic probability transform [20] provides the link between the uncertainty representations on 2^{Ω} and on Ω (as in probability theory):

$$\forall H \in \Omega, BetP(H) = \sum_{A \in 2^{\Omega} \setminus \{\emptyset\} / H \in A} \frac{m(A)}{|A|(1 - m(\emptyset))}, \tag{1}$$

where $|A|$ denotes the cardinality of hypothesis A.