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Adaptive H_∞ control in finite frequency domain for uncertain linear systems

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ABSTRACT

This paper is concerned with the problem of adaptive H_∞ controller design in finite frequency domains for uncertain linear systems. The uncertainties are assumed to be time-invariant, unknown, but bounded, which appear affinely in the matrices of system models. An adaptive mechanism is introduced to construct a novel finite frequency H_∞ controller with time-varying gains. By using Lyapunov theory and Parseval's Theorem, the controller design conditions are given in terms of a set of linear matrix inequalities (LMIs). It is shown that the proposed finite frequency controller with time-varying gains can achieve better H_∞ performance than the traditional ones with fixed gains. Finally, a numerical example of the F-18 aircraft model is given to illustrate the presented theoretical results.

1. Introduction

In many applications, modelling errors and system uncertainties in plant models are inevitable. To achieve satisfactory system performances, robust control problems have received increasing attention during the past decades [7]. An effective tool for robust controller designs is based on the use of LMI techniques [9], which are computationally simple and numerically reliable.

Within this framework, a number of controller design approaches have been given in the literature. In [11], a robust controller was designed for the uncertain time-invariant (LTI) retarded system. In [34], the robust H_∞ controller design for uncertain stochastic nonlinear systems was considered. In [35], the $L_2 - L_\infty$ controller design was investigated for uncertain two dimensional systems. In [31], a fuzzy adaptive backstepping design procedure was proposed for a class of uncertain nonlinear systems. In [16], the problem of insensitive tracking control for a class of complex networked systems with controller additive coefficient variations was addressed. In [32], the state observer-based adaptive fuzzy control techniques were developed. Note that the norm bounded uncertainties have been widely studied in the above results. For the polytopic type uncertainties, there also exist various controller design approaches. In [1], the problem of robust H_∞ dynamic output feedback controller design for uncertain continuous-time linear systems was addressed. The delay-dependent robust H_∞ filtering for uncertain discrete-time singular systems was studied in [17]. An approach was provided in [4] to design robust dynamic output feedback controller for linear systems with polytopic uncertainties. In [39], the problems of robust stability and stabilization were investigated for a class of continuous-time uncertain systems. In [6], the problem of robust static output feedback control was studied for uncertain continuous-time linear systems. In [29,2], the authors considered the problems of

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robust controller designs of linear discrete-time periodic systems and linear switched systems with polytopic-type uncertainties. In [33], a robust control storage function method was developed for solving the H_∞ control problem of single-input polytopic nonlinear systems.

It should be pointed out that, although the stability and robust H_∞ performance of the closed-loop systems can be ensured by using the above methods, the designed controllers with fixed gains may lead to conservatism when large parameter uncertainties are encountered. Based on this observation, the controllers and filters with varying gains have been designed in [37,36,23,3,24–26,19] by using adaptive or switching techniques. The comparison results have shown the superiority of the adaptive controllers and filters with varying gains.

While adaptive H_∞ control approaches are more effective in addressing the large parameter uncertainties, the methods in [37,36] are not completely compatible with practical requirements, because they overlook a vital fact that external inputs may belong to known finite frequency ranges, which include low/middle/high frequency (LF/MF/HF) ranges. For instance, the information of incipient faults is always contained within low frequency bands as the fault development is slow [20]; in addition, the frequency ranges of reference inputs generally lie in known finite intervals. For these finite frequency external inputs, the full frequency controller design approaches will be much conservative due to overdesign.

Recently, a milestone in the road of investigating finite frequency specifications of LTI systems is the generalized Kalman–Yakubovich–Popov (GKYP) lemma developed in [15,14], where the equivalence between a frequency domain inequality and an LMI over a finite frequency range was established. Based on the generalized KYP lemma, a number of controller and filter design results have been derived, allowing designers to impose different performance requirements over chosen finite frequency ranges. In [38], the low frequency positive real control was considered for delta operator systems, and the generalized KYP lemma was used in [27] to develop new stability tests for differential linear repetitive processes. In [5], the model reduction problem of two-dimensional digital filters over finite frequency ranges was investigated. In [18,30], the finite frequency H_∞ controllers were designed for LTI systems. In [10,21], the finite frequency H_∞ filters were designed for linear time-delayed and two dimensional discrete-time systems, respectively. The advantages of finite frequency controllers and filters have been proven in [18,30,38,27,10,5,21]. However, up to now, the design problem of adaptive finite frequency controller has not been solved. A key challenge is that the generalized KYP lemma [15,14] cannot be used for the control systems with adaptive mechanism, which are time-varying in nature. Therefore, it is necessary to develop a new approach to resolve this problem, which motivates the present investigation.

This paper is concerned with the problem of adaptive finite frequency controller design for uncertain linear systems. The uncertainties are assumed to be time-invariant, unknown, but bounded, which appear affinely in the matrices of system models. An adaptive mechanism is introduced to construct a robust H_∞ controller with varying gains. Instead of using generalized KYP lemma, the problem of finite frequency performance analysis is studied via the Parseval's Theorem [40] and Lyapunov function approach. Towards this direction, the controller design conditions are then given in terms of a set of LMIs. It is shown that the proposed adaptive finite frequency controller with varying gains can achieve better H_∞ performance than the traditional finite frequency controllers with fixed gains.

The rest of the paper is organized as follows: the problem statement and preliminaries are presented in Section 2. The problem of finite frequency performance analysis is summarized in Section 3. The finite frequency controller design conditions are provided in Section 4. In Section 5, the effectiveness and advantages of the proposed method are illustrated by a numerical example of the F-18 aircraft model, and some conclusions are given in Section 6.

Notations: $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; I represents the identity matrix. In block symmetric matrices or long matrix expressions, we use a star $(*)$ to represent a term that is induced by symmetry. A block diagonal matrix with matrices $X_1, X_2, \ldots, X_n$ on its main diagonal is denoted as $\text{diag}\{X_1, X_2, \ldots, X_n\}$. For a matrix A, its complex conjugate transpose is denoted by A^*, $He(A) =: A + A^*$, $\mathbf{He}(A) =: \frac{A+A^*}{2}$. For a symmetric matrix, $A > 0 (\geqslant 0)$ and $A < 0 (\leqslant 0)$ denote positive (semi) definiteness and negative (semi) definiteness. L_2 denotes the Hilbert space of square integrable functions with the following norm: $\|v(t)\|_2 = \{\int_0^\infty v^*(t)v(t)dt\}^{\frac{1}{2}}$.

2. Problem statement and preliminaries

2.1. System model

Consider the following uncertain linear system

$$\begin{aligned} \dot{x}(t) &= A(\theta)x(t) + B(\theta)u(t) + B_d(\theta)d(t) \\ z(t) &= C(\theta)x(t) + D(\theta)u(t) \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state space vector, $z(t) \in \mathbb{R}^p$ is the performance output, $d(t) \in L_2$ denotes the external input and the frequency of $d(t)$ resides in a known but finite frequency set Ω, and $u(t)$ is the control input. Here

$$\begin{aligned} &A(\theta) = A_1 + \sum_{i=2}^{N} \theta_i A_i, B(\theta) = B_1 + \sum_{i=2}^{N} \theta_i B_i, B_d(\theta) = B_{d1} + \sum_{i=2}^{N} \theta_i B_{di}, \\ &C(\theta) = C_1 + \sum_{i=2}^{N} \theta_i C_i, D(\theta) = D_1 + \sum_{i=2}^{N} \theta_i D_i \end{aligned} \tag{2}$$