Information Sciences 314 (2015) 41–54

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Characterizations of coverings for upper approximation operators being closure operators ☆

Xiaoxia Bian [a], Pei Wang [b,*], Zuoming Yu [c], Xiaole Bai [d], Bin Chen [e]

[a] Department of Fundamental Sciences, Yancheng Institute of Technology, Yancheng, Jiangsu 224051, PR China
[b] Department of Mathematics and Information Science, Yulin Normal University, Yulin, Guangxi 537000, PR China
[c] School of Zhangjiagang, Jiangsu University of Science and Technology, Zhangjiagang 215600, PR China
[d] Bed Bath & Beyond Inc, Union, NJ 07083, USA
[e] Hangzhou Xiaoshan Sixth Senior High School, Hangzhou 311200, PR China

ARTICLE INFO

Article history:
Received 7 June 2014
Received in revised form 20 March 2015
Accepted 27 March 2015
Available online 31 March 2015

Keywords:
Covering-based upper approximation operator
Closure operator
First symmetrical condition
Second symmetrical condition
Triangle chain condition

ABSTRACT

In this paper, we define two conditions of symmetry for the covering $\mathcal{C}$ in a covering-based approximation space $(U, \mathcal{C})$. By using these conditions and the triangle chain condition, we give general, topological and intuitive characterizations of the covering $\mathcal{C}$ for three types of covering-based upper approximation operators being closure operators. We also give descriptions of $(U, \mathcal{C})$ in terms of information exchange systems when these operators are closure operators. These results answer an open problem raised in Ge et al. (2012). As an example application of our characterizations, we discuss the problem of implications among conditions for different types of covering-based upper approximation operators being closure operators.

1. Introduction

The concept of rough set was originally proposed by Pawlak [17]. It is a mathematical tool for handling uncertain knowledge, which has been successfully applied in pattern recognition, data mining, machine learning, and so on [14,16,18,37]. Compared to fuzzy sets, which are used for a similar purpose, rough sets are more general/flexible tools. In Pawlak's rough set theory, partition or equivalence relation is explicitly used in the definitions of the lower and upper approximations. Such a partition or equivalence relation is too restrictive for many applications because it can only deal with complete information systems [16,18,38].

Generalizations of rough set theory were considered by scholars in order to address this issue in complex practical problems. One approach was to develop extensions on equivalence relations, e.g., to extend them to tolerance relations [26], similarity relations [27], ordinary binary relations [32], and others [29,44]. Another important approach was to relax the partition to a covering and then obtain the covering-based rough sets. Zakowski first generalized the classical rough set theory by using coverings of a universe instead of partitions [35]. Such generalization leads to various covering-based approximation operators that are of both theoretical and practical importance [3,4,6,7,11,8,15,21,24,25,28,30,31,40,41,45]. In the survey

☆ This paper is supported by NSFC (Nos. 61472469, 11471153, 11401262), BK20140503 and 2014GXNSFBA118015.
* Corresponding author.
E-mail addresses: xiaoxia.bian@163.com (X. Bian), elspwang@sina.com (P. Wang), yuzuoming1981@gmail.com (Z. Yu), xiaole.bai@gmail.com (X. Bai), chenbin031405@126.com (B. Chen).

http://dx.doi.org/10.1016/j.ins.2015.03.060
0020-0255/

papers [24,33], the covering-based approximation operators presented in rough-set literature and their properties were listed.

As data mining gets increasingly popular in recent years, the relationships between properties of covering-based approximation operators and their corresponding coverings have attracted intensive research [1,20,21,34,43,45–47,49,50]. It is worth noting that topological approaches have provided a valuable perspective and have also played an important role in rough set theory study [2,7,11,13,19,22,23,34,36,43]. Topological properties therefore have gotten a lot of attention [2,12,13,29,32,34,39,42,43]. In [46,48], W. Zhu and F. Wang discussed the relationship between properties of four types of covering-based upper approximation operators and their corresponding coverings. In [9], the problems on characterizations of coverings for these operators being closure operators were discussed in depth. Ge et al. gave not only general, but also topological characterizations of coverings for these operators being closure operators. Furthermore, they gave intuitive characterizations of covering and information exchange systems representation of covering-based approximation spaces when two of these operators are closure operators. Besides these operators, there were several other covering-based upper approximation operators listed in [24,33]. At the end of [9], Ge et al. raised the following question:

Question 1.1 (Question 9.4 of [9]). What are characterizations, either general, topological or intuitive, of a covering $\mathcal{C}$ for the covering-based upper approximation operators listed in [24] being closure operators? What kind of information exchange systems does a covering-based approximation space $(U, \mathcal{C})$ represent when any of them is a closure operator?

2. Background

We present some basic concepts that will be widely used in this paper. In the following discussion, unless it is mentioned specially, the universe of discourse U is considered finite. $P(U)$ denotes the family of all subsets of U. Suppose that $\mathcal{C} \subseteq P(U)$. If none of sets in $\mathcal{C}$ is empty, and $\bigcup \mathcal{C} = U, \mathcal{C}$ is called a covering of U. We call an ordered pair $(U, \mathcal{C})$ a covering-based approximation space. For every $X \subseteq U, \sim X$ denotes the complement set of X in U, i.e., $\sim X = U \setminus X$.

Definition 2.1. Let $\mathcal{C}$ be a covering of $U, x \in U$ and $X \in P(U)$. Then denote

1. $\widetilde{x} = \{y \in U : \forall K \in \mathcal{C}(x \in K \Longleftrightarrow y \in K)\}$;
2. $\underline{C}(X) = \bigcup\{C \in \mathcal{C} : C \subseteq X\}$;
3. $Friends(x) = \bigcup\{C \in \mathcal{C} : x \in C\}$;
4. $Md(x) = \{C \in \mathcal{C} : (x \in C) \wedge (\forall K \in \mathcal{C})(x \in K \wedge K \subseteq C \Rightarrow C = K)\}$;
5. $N(x) = \bigcap\{C \in \mathcal{C} : x \in C\}$;
6. $CFriends(x) = \bigcup Md(x)$.

Remark 1. Except for $Md(x)$, all other symbols defined above represent subsets of U, while $Md(x)$ denotes a subset of $P(U)$.

Definition 2.2 (Unary covering). Let $\mathcal{C}$ be a covering of U. Then $\mathcal{C}$ is called unary if $\forall x \in U, |Md(x)| = 1$.

We use $\overline{C_n}, 1 \leqslant n \leqslant 11$ to represent all the different types of covering-based upper approximation operators listed in [24] and only use these symbols in the sequel. Note that these operators were denoted by different symbols in [24] and other rough-set literature.

Definition 2.3. $\forall X \subseteq U$,

1. $\overline{C_1}(X) = \underline{C}(X) \cup \bigcup\{\bigcup Md(x) : x \in X \setminus \underline{C}(X)\}$;
2. $\overline{C_2}(X) = \bigcup\{C \in \mathcal{C} : C \cap X \neq \emptyset\}$;
3. $\overline{C_3}(X) = \bigcup\{\bigcup Md(x) : x \in X\}$;
4. $\overline{C_4}(X) = \underline{C}(X) \cup \bigcup\{C \in \mathcal{C} : C \cap (X \setminus \underline{C}(X)) \neq \emptyset\}$;
5. $\overline{C_5}(X) = \{y : \forall C(y \in C \Rightarrow C \cap X \neq \emptyset)\}$;
6. $\overline{C_6}(X) = \{x \in U : \forall u(u \in N(x) \rightarrow N(u) \cap X \neq \emptyset)\}$;
7. $\overline{C_7}(X) = \bigcup\{N(x) : N(x) \cap X \neq \emptyset\}$;
8. $\overline{C_8}(X) = \{z : \forall y(z \in Friends(y) \Rightarrow Friends(y) \cap X \neq \emptyset)\}$;
9. $\overline{C_9}(X) = \{x \in U : N(x) \cap X \neq \emptyset\}$;
10. $\overline{C_{10}}(X) = \bigcup\{N(x) : x \in X\} = \underline{C}(X) \cup \bigcup\{N(x) : x \in X \setminus \underline{C}(X)\}$;
11. $\overline{C_{11}}(X) = \bigcup\{\widetilde{x} : \widetilde{x} \cap X \neq \emptyset\}$.