Information Sciences 314 (2015) 169–183

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Consistency analysis and priority derivation of triangular fuzzy preference relations based on modal value and geometric mean

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ARTICLE INFO

Article history:
Received 3 August 2014
Received in revised form 22 March 2015
Accepted 30 March 2015
Available online 4 April 2015

Keywords:
Triangular fuzzy preference relation
Consistency
Geometric mean
Logarithmic least square
Multi-criteria decision analysis

ABSTRACT

Triangular fuzzy preference relation (TFPR) is an effective framework to model pairwise estimations with imprecision and vagueness. In order to obtain a reliable and rational decision result, it is important to investigate consistency and priority derivation of TFPRs. The paper analyzes existing definitions and properties of consistent TFPRs, and illustrates that they have no invariance with respect to permutations of decision alternatives. A new triangular fuzzy arithmetic based transitivity equation is introduced to define consistent TFPRs. The new transitivity equation reflects multiplicative consistency of modal values and multiplicative consistency of geometric means of triangular fuzzy estimations. Some properties are presented for consistent TFPRs, and a notion of acceptable consistency is put forward for TFPRs. Geometric mean and uncertainty ratio based transformation formulae are devised to convert normalized triangular fuzzy multiplicative weights into consistent TFPRs. A logarithmic least square model is further established for deriving a normalized triangular fuzzy multiplicative weight vector from a TFPR with acceptable consistency. A geometric mean based method is developed to compare and rank triangular fuzzy multiplicative weights. Three numerical examples including a group decision making problem are examined to demonstrate validity and advantages of the proposed models.

1. Introduction

Preference relations (or called pairwise comparison matrices) are practical and popular tools used to express decision-makers' pairwise estimations in multi-criteria decision analysis. The classical Analytic Hierarchy Process (AHP) [32] employs multiplicative preference relations to characterize ratio-based precise estimations provided by decision-makers. However, in many decision making problems, decision-makers' estimations are imprecise and vague [18,29]. As an extension of numeric values, the fuzzy set [50] is regarded as a rational framework for expressing vague and uncertain estimations. Based on granular computing [26], new methodologies have been developed to elicit fuzzy preference values from linguistic preference relations by using information granules in [7,9,27,29]. Linguistic terms are often associated with triangular fuzzy numbers [20]. Laarhoven and Pedrycz [37] introduced triangular fuzzy preference relations (TFPRs) whose elements are fuzzy numbers with triangular membership functions, and put forward the fuzzy priority theory. Since this seminal work, TFPRs and fuzzy AHP have been extensively studied [5,6,23,28,30,37,40,48,49,51] and triggered numerous successful applications [2,10,12,16,17,22,24,25,31,34,39,41,46].

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http://dx.doi.org/10.1016/j.ins.2015.03.074
0020-0255/

An active research topic is the derivation of priority weight vectors based on preference relations. A number of priority methods have been developed to derive fuzzy weights from TFPRs. For instance, Van Laarhoven and Pedrycz [37] developed a logarithmic least square method for obtaining triangular fuzzy weights of TFPRs. Xu [48] established a fuzzy least square model based on the distance metric. Buckley et al. [6] put forward a geometric mean method for deriving fuzzy weights from a trapezoidal fuzzy preference relation that is an extension of a TFPR. It should be noted that there are two different constraints employed to normalize a numeric weight vector $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$. One is the additive normalization constraint, i.e., $\sum_{i=1}^{n}\omega_i = 1$. The other is the multiplicative normalization constraint, namely, $\prod_{i=1}^{n}\omega_i = 1$. This multiplicative constraint is suggested in the multiplicative AHP [3,11,21,35,45]. As all estimations in TFPRs are expressed by ratio based triangular fuzzy numbers and the indifference between two alternatives is characterized by the triangular fuzzy number $(1,1,1)$, the normalized triangular fuzzy priority weights are assumed to be multiplicative in this paper.

Another key issue is consistency of preference relations [1,7,8,14,32,36,42,47]. The consistency usually refers to contradiction-free estimations and is reflected through transitivity among three or more pairwise estimations [42]. This concept has invariance to permutations of decision alternatives, which is one of the five axiomatic properties introduced by Brunelli and Fedrizzi [4] for characterizing inconsistency indexes of pairwise comparisons. An appropriate transitivity constraint would be favorable to the derivation of rational and reliable priority weights from the corresponding preference relations. Moreover, such transitivity constraints are often used to study acceptability of the decision-maker's pairwise estimations. Many researchers have paid attention to the consistency study for different kinds of preference relations. The research can be roughly classified into two categories. One is to define transitivity constraints by mathematic equations [5,14,15,32,36,42–44,47], while the other is based on feasible region models [19,33,38,45]. Saaty [32] gave a multiplicative transitivity equation to define consistency of multiplicative preference relations, and put forward the consistency index (*CI*) and the consistency ratio (*CR*) to measure the inconsistency level of a multiplicative preference relation. Buckley [5] introduced fuzzy-based equations to define consistent trapezoidal fuzzy preference relations and consistent TFPRs by directly fuzzifying Saaty's transitivity equation. Recently, Dubois [13] pointed out that a consistent TFPR is technically nonexistent according to the fuzzy-based transitivity equation proposed by Buckley [5]. Wang and Chen [40] proposed additively consistent properties of TFPRs and applied them to improve consistency of fuzzy AHP. Liu et al. [23] presented a new consistency definition of TFPRs by constructing three multiplicative preference relations, and put forward some properties of consistent TFPRs. However, a close analysis in Section 3 shows that the definitions and properties of consistent TFPRs in [23,40] have no invariance with respect to permutations of decision alternatives, and their applications may lead to conflict results.

The paper focuses on consistency and acceptability of TFPRs as well as deriving normalized triangular fuzzy multiplicative weight vectors from TFPRs. We first analyze the definitions and properties of consistent TFPRs by Liu et al. [23] and Wang and Chen [40], and illustrate that they are flawed by not being invariance to permutations of decision alternatives. A new consistency definition is put forward for TFPRs. Some useful properties are supplied for consistent TFPRs, and acceptable consistency of TFPRs is then introduced to measure the inconsistency level of triangular fuzzy estimations. A geometric mean based difference index is defined to measure the difference level between two TFPRs. Subsequently, we devise geometric mean and uncertainty ratio based formulae to transform normalized triangular fuzzy multiplicative weights into consistent TFPRs. Based on these transformation formulae, multi-objective logarithmic least square models are established for deriving priority weights from TFPRs. By introducing auxiliary equality constraints, a logarithmic least square model is then developed to obtain a normalized triangular fuzzy multiplicative weight vector from a TFPR. Finally, a geometric mean based formula is presented to compare and rank normalized triangular fuzzy multiplicative weights.

The rest of this paper is organized as follows. Section 2 reviews main concepts and results relating to Saaty's consistency of multiplicative preference relations, arithmetic operations of triangular fuzzy numbers, and TFPRs. Section 3 analyzes existing definitions and properties of consistent TFPRs. A new consistency definition is introduced and some properties of consistent TFPRs are given in Section 4. Section 5 develops a priority method to derive normalized triangular fuzzy multiplicative weights from TFPRs. Section 6 presents three illustrative examples including a group decision making problem with TFPRs. The main conclusions are provided in Section 7.

2. Preliminaries

In the classical AHP, pairwise comparisons on a decision alternative set $X = \{x_1, x_2, \ldots, x_n\}$ are expressed by a multiplicative preference relation $A = (a_{ij})_{n\times n}$ such that

$$1/S \leqslant a_{ij} \leqslant S, \quad a_{ij}a_{ji} = 1, \quad a_{ii} = 1, \qquad i,j = 1,2,\ldots,n \tag{2.1}$$

where a_{ij} is a ratio-based estimation chosen from a bounded scale $[1/S, S]$, and indicates the decision-maker's preference intensity for the alternative x_i over x_j.

We say that a multiplicative preference relation $A = (a_{ij})_{n\times n}$ is consistent if

$$a_{ij} = a_{ik}a_{kj}, \quad i,j,k = 1,2,\ldots,n \tag{2.2}$$

If there exist differences between a_{ij} and $a_{ik}a_{kj}$ for some $i, j, k = 1, 2, \ldots, n$, then A is said to be inconsistent.

By applying the multiplicative reciprocity of $a_{ij}a_{ji} = 1, \forall i,j = 1,2,\ldots,n$, Eq. (2.2) can be equivalently expressed as

$$a_{ij}a_{jk}a_{ki} = a_{ik}a_{kj}a_{ji}, \quad i,j,k = 1,2,\ldots,n. \tag{2.3}$$