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Generalized convexity in fuzzy vector optimization through a linear ordering ☆

M. Arana-Jiménez [a], A. Rufián-Lizana [b], Y. Chalco-Cano [c,*], H. Román-Flores [c]

[a] Department of Statistics and Operational Research, University of Cádiz, Spain
[b] Department of Statistics and Operational Research, University of Sevilla, Spain
[c] Instituto de Alta Investigación, Universidad de Tarapacá, Casilla 7D, Arica, Chile

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ABSTRACT

In this article we study efficiency and weakly efficiency in fuzzy vector optimization. After formulating the problem, we introduce two new concepts of generalized convexity for fuzzy vector mappings based on the generalized Hukuhara differentiability, pseudoinvexity-I and pseudoinvexity-II. We prove that pseudoinvexity is the necessary and sufficient condition for a stationary point to be a solution of a fuzzy vector optimization problem. We give conditions to insure that a fuzzy vector mapping is invex and pseudoinvex (I and II). Moreover, we present some examples to illustrate the results. Lastly, we use these results to study the class of problems which have uncertainty and inaccuracies in the objective function coefficients of mathematical programming models.

1. Introduction

Since the emergence of fuzzy set theory in the 60s, several topics have been developed and, in particular, the area of fuzzy optimization has grown in recent years. The collection of papers on fuzzy optimization edited by Słowiński [41,42], Delgado et al. [13], Lodwick and Kacprzyk [29] and the books of Lai and Hwang [26,27] contain a review of this topic from a very broad point view.

Convex analysis is a fundamental tool in optimization theory and is a natural one to be applied to fuzzy optimization. The properties of convexity and fuzzy optimization and related problems attracted a wide range of research [4,11,10,19,20,23,32,37–40,46–53,55].

In [20] the authors proposed a linear ordering on the space of fuzzy intervals. Now, for each fuzzy mapping (fuzzy interval-valued mapping) F, based on this linear ordering, they introduced a ranking function (real-valued function) T_F on the domain of the fuzzy mapping F. The concepts of convexity and quasiconvexity for a fuzzy mapping F through the real-valued function T_F were excellently presented in the article [52]. In [39] the Weirstrass theorem was extended from real-valued function to the fuzzy mappings context. Also, the local–global minimum properties of convex fuzzy mappings were obtained. More recently, the concept of invex fuzzy mapping and necessary and sufficient conditions for scalar fuzzy optimization were obtained in [8].

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* Corresponding author. Tel./fax: +56 58 2230334.
E-mail address: ychalco@uta.cl (Y. Chalco-Cano).

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This article formulates fuzzy vector optimization. For this, we generalize the linear ordering to the space of fuzzy vector set and we present the concept of solution for the fuzzy vector optimization problem. Then, we give necessary optimality conditions for this class of problems through the ranking function (vector-valued function) T_F. Later, we extend the concept of convex fuzzy mapping to the class of invex, pseudoinvex-I and pseudoinvex-II fuzzy vector mappings. Toward this end, we consider the concept of generalized Hukuhara differentiable fuzzy vector mapping, which is the more general concept of differentiability existing in the literature for these classes of mappings [2,3,6,7,25]. Using the new classes of fuzzy vector mappings, that is, pseudoinvex-I and pseudoinvex-II fuzzy vector mappings, we obtain some characterizations of efficient and weakly efficient solutions for fuzzy vector problems in relation to the new optimality condition of stationary point introduced in this article. We give conditions for fuzzy vector mappings to assure that it is invex and pseudoinvex (I and II). These conditions are based on the invexity of the sum of the endpoint functions of a fuzzy vector mapping. Finally, to illustrate the results we obtained, we present several mathematical programming problems where the coefficients of the objective functions are considered as being fuzzy sets.

2. Notation and the space of fuzzy intervals

A fuzzy set u on $\mathbb{R}^n$ is a mapping $u : \mathbb{R}^n \to [0,1]$. For each fuzzy set u, we denote its α-level set as $[u]^\alpha = \{x \in \mathbb{R}^n | u(x) \geqslant \alpha\}$ for any $\alpha \in (0,1]$. The support of u we denote by $supp(u)$ where $supp(u) = \{x \in \mathbb{R}^n | u(x) > 0\}$. The closure of $supp(u)$ defines the 0-level of u, i.e. $[u]^0 = cl(supp(u))$ where $cl(M)$ means the closure of the subset $M \subset \mathbb{R}^n$.

Let $\mathcal{F}_C$ denote the family of all fuzzy intervals. So, for any $u \in \mathcal{F}_C$ we have that $[u]^\alpha \in \mathcal{K}_C$ for all $\alpha \in [0,1]$, where $\mathcal{K}_C$ denotes the family of all closed and bounded intervals in $\mathbb{R}$. Thus, the α-levels of a fuzzy interval are intervals which we denote by $[u]^\alpha = [\underline{u}_\alpha, \overline{u}_\alpha]$, $\underline{u}_\alpha, \overline{u}_\alpha \in \mathbb{R}$ for all $\alpha \in [0,1]$. If $[u]^1$ is a singleton then we say that u is a fuzzy number. Triangular fuzzy numbers are a special type of fuzzy numbers which are defined by three real numbers $a \leqslant b \leqslant c$ and we write $u = (a,b,c)$ and

$$[u]^\alpha = [a + (b-a)\alpha, c - (c-b)\alpha],$$

for all $\alpha \in [0,1]$. Also we can denote a triangular fuzzy number $u = (a,b,c)$ by $\tilde{b}$.

The well-known characterization theorem makes the connection between a fuzzy interval and their endpoint functions [20].

Theorem 1 [20]. *Let u be a fuzzy interval. Then the functions $\underline{u}, \overline{u} : [0,1] \to \mathbb{R}$, defining the endpoints of the α-level sets of u ($\underline{u}(\alpha) = \underline{u}_\alpha$ and $\overline{u}(\alpha) = \overline{u}_\alpha$), satisfy the following conditions:*

(i) *$\underline{u}$ is a bounded, non-decreasing, left-continuous function in $(0,1]$ and it is right-continuous at 0.*
(ii) *$\overline{u}$ is a bounded, non-increasing, left-continuous function in $(0,1]$ and it is right-continuous at 0.*
(iii) $\underline{u}(1) \leqslant \overline{u}(1)$.

Reciprocally, given two functions that satisfy the above conditions they uniquely determine a fuzzy interval.

For fuzzy intervals $u, v \in \mathcal{F}_C$ and for any real number λ, the addition $u + v$ and scalar multiplication λu are well known, and we refer the reader to see [14,31]. Also, given two fuzzy intervals u, v, we denote by $u \ominus_{gH} v$ the generalized Hukuhara difference (gH-difference for short) between u and v (see [44]).

Given $u, v \in \mathcal{F}_C$, we define the distance between u and v by

$$D(u,v) = \sup_{\alpha \in [0,1]} \max\{|\underline{u}_\alpha - \underline{v}_\alpha|, |\overline{u}_\alpha - \overline{v}_\alpha|\}.$$

$(\mathcal{F}_C, D)$ is a complete metric space (see [14]). We denote by $\mathcal{F}_C^C$ the family of all level-continuous fuzzy intervals [36]. It is well known that $(\mathcal{F}_C^C, D)$ is a separable and complete metric space [36]. Moreover, $\mathcal{F}_C^C$ is a closed subspace of $\mathcal{F}_C$. Further, if $u = (\underline{u}, \overline{u})$ be a fuzzy interval, then, $u \in \mathcal{F}_C^C$ if and only if $\underline{u}$ and $\overline{u}$ are continuous functions (see [8]).

2.1. A linear ordering on the space of fuzzy intervals

Many authors have studied different methods for ranking fuzzy intervals. Most of these authors suggest mapping each fuzzy interval into the real line to define a ranking function, see for instance [15–17,45,54]. In [15] a ranking function called the Average Value (A.V.) was introduced. The A.V. was defined as dependent on several parameters, allowing flexibility in the final classification. The following definition of ranking function, introduced by Tsumura et al. in [45], is a particular case of A.V. considering a mean optimism degree. For more details see [5,15].