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Positive filtering for positive Takagi–Sugeno fuzzy systems under ℓ_1 performance

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ABSTRACT

In this paper, the positive filtering problem is addressed for positive Takagi–Sugeno (T–S) fuzzy systems under the ℓ_1-induced performance. To estimate the output of positive T–S fuzzy systems, error-bounding positive filters are constructed. A new performance characterization is first established to guarantee the asymptotic stability of the filtering error system with the ℓ_1-induced performance. Moreover, sufficient conditions expressed by linear programming problems are derived to design the required filters. Finally, a numerical example is presented to show the effectiveness of the derived theoretical results.

1. Introduction

Positive systems can be found in different application fields such as physical, engineering and social sciences [18,33]. The variables of positive systems are non-negative because they denote the concentrations or amounts of material in application fields. Positive systems have special structures and possess many unique features. Therefore, many new problems appear and some previous approach used for general systems are no longer applicable to positive systems. In recent years, such systems have been studied in the literature [15,22,27,41,43,47,48,50,51,35]. For example, for a given transfer function, a positive state-space representation has been proposed in [2]. The state-feedback controller has been designed and moreover the controller synthesis results have been expressed by the linear matrix inequality (LMI) problems and the linear programming problems in [1,16]. The problem of controllability and reachability has been investigated for positive systems in [13,34,42]. Stability analysis results for compartmental dynamic systems has been presented in [17,20]. Positive observer has been designed for positive systems in [25]. In [14], the model reduction problem for positive systems has been solved. In addition, the analysis and synthesis problems for some special positive systems have been addressed, like positive systems with time delays [28,49,29,36,31,53] and 2-D positive systems [22,24].

We note that existing research has been conducted mainly for positive linear systems and actually nonlinearities commonly exist in many practical systems. Nonlinearity is difficult to tackle and some results derived for linear positive systems cannot be directly used for nonlinear positive systems. A nonlinear system can be approximated by the T–S fuzzy

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model, which provides an efficient method to tackle some problems for nonlinear systems [21,37–40,45,52]. Through the modeling approach, nonlinear systems can be transformed into a framework composed of subsystems. As a result, some research approaches applicable to linear systems can be used for nonlinear systems. Many important results have been reported on fuzzy systems, for example, a novel approach was proposed in [46] for stability analysis and stabilization of discrete-time T–S fuzzy delay systems. As a result of the novel idea of delay partitioning technique, the proposed stability conditions in [46] are much less conservative than most of the existing results. A fuzzy filter design approach was established for fuzzy stochastic systems in [44], and the proposed approach can be used for fault detection problem due to its strong robustness and less conservativeness. Recently, many authors have focused their interest on positive T–S fuzzy systems [4]. In detail, the sufficient conditions of asymptotic stability and stabilization for discrete-time positive T–S systems are proposed in [4]. In [32], the problem of stability and constrained control is addressed for discrete-time T–S fuzzy positive systems with time-varying delays. The problem of stabilization by state feedback control of Takagi–Sugeno (T–S) fuzzy discrete-time systems with multiple fixed delays is dealt with in [3].

On the other hand, it is noted that previous approaches derived for the filtering problem of general systems cannot be directly used for positive systems. The reason lies in that these approaches cannot guarantee the positivity of the filter. The constraint that the filter should be positive makes the filtering problem for positive systems more complicated and cannot be easily tackled with existing approaches. It should be mentioned that in [26], a positivity preserved filter is designed for positive systems. Nevertheless, the design of positivity preserved filter for positive systems is still an open problem and remains challenging. In addition, most of the existing results about positive systems were derived with the quadratic Lyapunov function and correspondingly many results are treated under the LMI framework [5]. In recent years, many researchers derived some results based on the linear Lyapunov function [1,6,8–12,18,19]. By using a linear Lyapunov function, a new approach is proposed to investigate the filtering problem for positive systems. Moreover, there are many different performances used in previous studies, such as H_∞ control [7], H_2/H_∞ control and H_2 control. It is noted that some frequently used performance measures such as H_∞ norm are based on the ℓ_2 signal space [23,30]. In some situations, these performance measures induced by ℓ_2 signals are not very natural to describe some of the features of practical physical positive systems. 1-norm can provide a more useful description for positive systems because 1-norm gives the sum of the values of the components, which is more appropriate, for instance, if the values represent the amount of material or the number of animal in a species. Based on the above discussion, we know that although many problems on positive systems have been studied, the synthesis problems for positive T–S fuzzy systems have not been fully investigated. Moreover, ℓ_1-induced performance has not been used in the study of positive T–S fuzzy systems. This motivates our study.

In this paper, the problem of ℓ_1-induced filtering is investigated for positive T–S fuzzy systems and the positivity is preserved in the filters. The main contributions of the paper are: (1) An ℓ_1-induced performance index is explicitly presented for fuzzy positive systems and analytically characterized under a linear Lyapunov function framework; (2) a pair of ℓ_1-induced fuzzy positive filters is first proposed such that the output of positive T–S fuzzy systems is estimated; and (3) sufficient conditions are established in terms of linear programming problems to design the desired positive filters.

The rest of this paper is organized as follows. Some important preliminaries are introduced in Section 2. The problem investigated in this paper is formulated in Section 3. In Section 4, the positive filter design procedure is proposed for positive T–S fuzzy systems. An example is provided in Section 5 to show the application of the theoretical results. The results are finally concluded in Section 6.

2. Preliminary results

In this section, we introduce notations and several results concerning positive fuzzy linear systems.

Let $\mathbb{R}$ denote the set of real numbers; $\mathbb{R}^n$ is the n-column real vectors; $\mathbb{R}^{n\times m}$ denotes the set of all real matrices of dimension $n \times m$. For a matrix $A \in \mathbb{R}^{m\times n}$, $[A]_{ij}$ denotes the element located at the ith row and the jth column; $[A]_{r,i}$ and $[A]_{c,j}$ denote the ith row, and the jth column, respectively. $A \geqslant\geqslant 0$ (respectively, $A \gg 0$) means that for all i and j, $[A]_{ij} \geqslant 0$ (respectively, $[A]_{ij} > 0$). The notation $A \geqslant\geqslant B$ (respectively, $A \gg B$) means that the matrix $A - B \geqslant\geqslant 0$ (respectively, $A - B \gg 0$). Let $\overline{\mathbb{R}}^n_+$ denote the nonnegative orthants of $\mathbb{R}^n$. The superscript "T" represents matrix transpose. $\|\cdot\|$ denotes the Euclidean norm for vectors. The 1-norm of a vector $x(k) = (x_1(k), x_2(k), \ldots, x_n(k))$ is defined as $\|x(k)\|_1 \triangleq \sum_{i=1}^n |x_i(k)|$. The induced 1-norm of a matrix $Q \triangleq [q_{ij}] \in \mathbb{R}^{m\times n}$ is denoted by $\|Q\|_1 \triangleq \max_{1\leqslant j\leqslant n}(\sum_{i=1}^m |q_{ij}|)$. The ℓ_1-norm of an infinite sequence x is defined as $\|x\|_{\ell_1} \triangleq \sum_{k=0}^\infty \|x(k)\|_1$. The space of all vector-valued functions defined on $\overline{\mathbb{R}}^n_+$ with finite ℓ_1 norm is denoted by $\ell_1(\overline{\mathbb{R}}^n_+)$. If the dimensions of matrices are not explicitly stated, it is assumed that the matrices have compatible dimensions for algebraic operations. Vector $\mathbf{1} = [1, 1, \ldots, 1]^T$.

Consider the following fuzzy system described by the ith rule as follows:

Model rule i: IF $\theta_1(k)$ is M_{i1} and $\theta_2(k)$ is M_{i2} and … and $\theta_g(k)$ is M_{ig}, THEN

$$\begin{cases} x(k+1) = A_i x(k) + B_{wi} w(k), \\ y(k) = C_i x(k) + D_{wi} w(k), \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^n$, $w(k) \in \mathbb{R}^m$ and $y(k) \in \mathbb{R}^q$ denote the system state, disturbance input and output, respectively. $i = 1, 2, \ldots, r$, is the rule number and $\theta_1(k), \theta_2(k), \ldots, \theta_g(k)$ are the premise variables. $M_{ie}(i = 1, 2, \ldots, r;\ e = 1, 2, \ldots, g)$ represents the fuzzy sets. Then, we have the final fuzzy system: